States and Mafias*

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PRELIMINARY

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Abstract

We analyze a model of the relationship between a government, a mafia, and a Firm, in which the government and mafia compete with one another to provide revenue protection services to the Firms.

1 Introduction

The enforcement of contractual arrangements is a essential to modern economic activity. This service is generally regarded as one of the essential public goods provided by governments. However, in many societies, governmental weakness and corruption create a situation in which extra-governmental organizations, such as Mafias, compete with governments to provide contract enforcement and revenue protection (Gamebetta 1994, Vareses 2000). The emergence of a strong Mafia on which the smooth functioning of the economy depends can

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introduce important forms of inefficiency and corruption into society. Understanding why such Mafias emerge, the extent to which voters and governments can prevent the development of a corrupt economy, and how the possibility of extortion affects economic performance is essential to the analysis of the political economy of transitions (Hay and Shleifer 1998).

We present a model of the relationship between the three key types of players in transition economies: governments, mafias, and two economically productive agents (Firms). The mafia is understood as an alternative provider of contract enforcement and revenue protection, rather than simply as an extortionist or thief. The government is modeled as having certain law enforcement responsibilities that it cannot fully avoid. However, the government can act corruptly by misappropriating tax revenues intended for law enforcement. The Firms endogenously choose the level of taxation, whether or not to hire the Mafia, and whether to reelect the government.

The Firms face a commitment problem. In each period, they must each choose whether to depend on the government for contract enforcement or whether to hire the Mafia for a fee. The commitment problem arises because, if only one Firm hires the Mafia, that Firm may be able to use the Mafia to extort money from the other Firm. If both Firms hire the Mafia, however, this cannot happen. The Firms prefer for neither to hire the Mafia than for both to hire the Mafia, in order to avoid paying fees, but they do not trust one another. It is here that the government comes in.

The Firms, acting as voters, endogenously choose how well to fund the government through taxation. They are willing to pay taxes because law enforcement can solve the commitment problem. Government law enforcement sometimes makes illegal activity so costly that it drives the Mafia out of business. Even when the Mafia is entirely eliminated, law enforcement can reduce the fess the Mafia is able to demand. Of course, the weakening of the Mafia comes at a price to the Firms—taxation. Moreover, the Firms also have to worry about government corruption—the government may expropriate resources rather than spend them on law enforcement. The Firms use electoral incentives to try to solve this moral hazard problem.

These trade-offs drive the key results in our model. We characterize an equilibrium in which, on the equilibrium path, it is possible for a dominant Mafia or a dominant government to emerge. If, in equilibrium, the Government emerges as dominant, this is because the Firms are willing to fund it through taxation and the electoral incentives are strong enough to limit government corruption. There are also corrupt equilibria in which the Firms conclude that it is more efficient not to fund the government and, thereby, allow the Mafia to dominate the economy. Finally, there are intermediate equilibria in which the Firms sometimes rely on the Government and sometimes hire the Mafia.

The model also provides insight into what sort of policy remedies are likely to move a country from a corrupt to an uncorrupt equilibrium. In particular, we demonstrate that increasing funding to the government will not always increase law and order or diminish Government corruption. This is because the government, in choosing how much to invest in law enforcement, balances two types of incentives. On the one hand, the government is tempted to expropriate tax revenues. On the other hand, the government has electoral incentives to invest in law enforcement. These electoral incentives come from the threat, by the Firms, not to reelect the government should they fail to challenge and defeat the Mafia. The Government's optimal level of corruption balances these interests.

2 The Extant Literature

To be added.

3 The Setup

There are four players: two Firms, the Mafia, and a Government official.[1] The sequence of play is as follows. At time 0, Firms choose the tax rate $\tau \in [0, 1]$. This choice is followed by two periods, $t = 1, 2$, with the game ending at the end of the second period. The Firms have a contractual relationship, the total value of which in each period is normalized to 1. At the beginning of period $t = 1$, Nature chooses the division of the benefits of that period's contract, and determines which Firm (called "Firm 1" or F1) gets proportion $\alpha^t \in (0, \frac{1}{2}$ of those benefits, and which Firm ("Firm 2" or F2) gets $(1 - \alpha^t)$. Each period a new division of the contract is selected, so which of the two Firms is favored (i.e., $F1$) can change from period to period. Generally, we use subscripts 1 and 2 to indicate a reference to F1 and F2, respectively, and superscripts to refer to the periods. Both the Firms and the Mafia observe Nature's selections, but the government does not. In the next event of the period, the Government official chooses a proportion of the tax revenue collected to commit to law enforcement, $\lambda^t \in [0, 1]$. Neither the Firms nor the Mafia observe λ^t. Next, the Mafia sets the fees ($\boldsymbol{\phi}^t = (\phi_1^t, \phi_2^t)$) that each Firm must pay for its services and makes a take-it-or-leave-it offer to each Firm of the corresponding fee. Notice that we allow the Mafia to price discriminate between the two Firms. Following these offers, Firms simultaneously choose whether to hire the Mafia. When a Firm hires the Mafia, it pays the fee regardless of outcome.

Let μ_{Fi}^t be the probability that $Fi = \{F1, F2\}$ accepts the Mafia's offer in t. If one Firm refuses the Mafia's offer while the other accepts, the former Firm appeals to the Government,

[1]As will be seen below, because the equilibrium strategies are independent of the connections between Firms' identities across periods, the game we study is consistent with an interpretation whereby each period's Firms are different. Similarly, because Firms' equilibrium choices are identical, we can also think of the present model as being one with any number of randomly matched Firms with the equilibrium we solve for interpreted as the symmetric equilibrium of the corresponding game.

which then challenges the Mafia. If both Firms accept the Mafia's offers, the Government official must *choose* whether to challenge the Mafia or not. Let γ be probability that the Government challenges the Mafia, given that both Firms hire the Mafia. The idea underlying these assumptions is that, if just one Firm hires the Mafia, the Government is obliged to attempt to provide that Firm with protection from extortion. However, if both Firms hire the Mafia, then the Government has the option of whether or not to engage in a fight against corruption in the economy.

The probability that the Government defeats the Mafia when they are in conflict is a function of the Government's level of investment in law enforcement ($\lambda\tau$). We denote this $Pr(G^t|\lambda^t\tau)$ and assume that it is increasing and concave in $\lambda^t\tau$. We will also frequently refer to the probability that the Mafia wins $Pr(M^t|\lambda^t\tau) = 1 - Pr(G^t|\lambda^t\tau)$.

If there is a conflict between the Mafia and Government, and the Mafia loses, the Mafia bears a cost k, which we interpret as the punishment imposed by the government. [2] The winner of this conflict determines the division of post-tax benefits $(1-\tau)$ between the Firms. If the Government wins, it imposes the outcome that corresponds to the actual realization of the contract, i.e., $(\alpha(1-\tau), (1-\alpha)(1-\tau))$. If the Mafia wins, it also imposes the actual realization if it was hired by both Firms. However, if only one Firm hired the Mafia, and the Mafia wins, it gives the entire surplus $(1-\tau)$ to the Firm that hired it. The idea, here, is that Government enforcement is essentially fair, reflecting the agreed upon contract. A Firm hires the Mafia to try to extort more than its rightful share from the other Firm. If both Firms hire the Mafia, this has an offsetting effect—neither Firm can extort from the other.[3] If only one Firm hires the Mafia, and this Mafia is able to prevail over Government law enforcement, then extortion occurs. We assume that, although the action of the Government challenging

[2]We assume that the Mafia's activities, including fighting the government, are financed by the fees paid by the Firms, and that the Mafia's claims of its ability corresponding to the informational assumptions of the model are "credible" - guaranteed by some background repeated interaction that generates the Mafia's reputation.

[3]We justify this assumption in more detail later in the analysis.

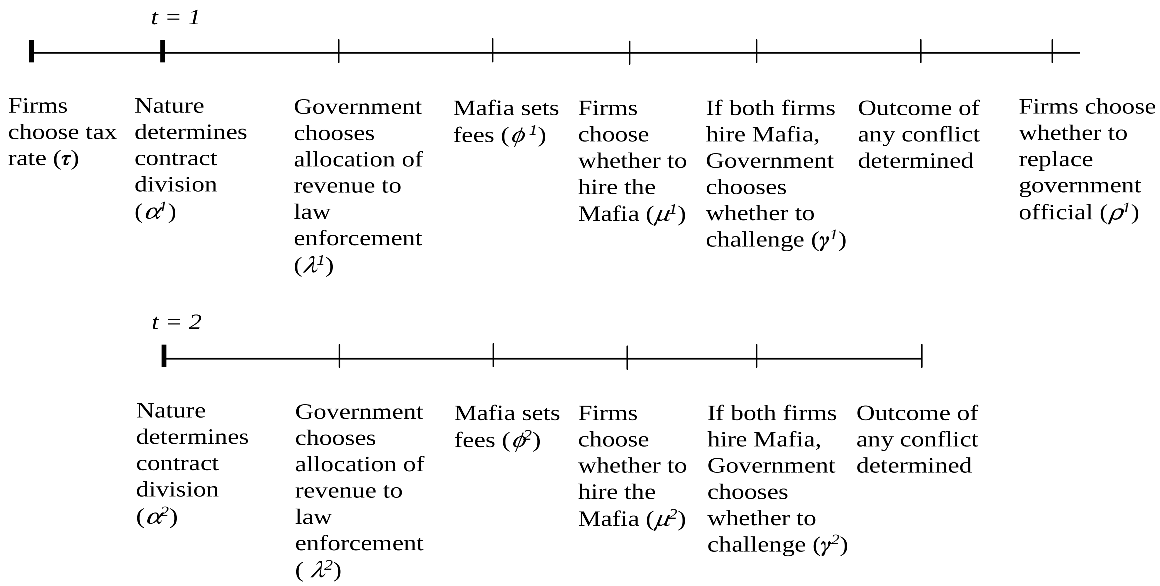

Figure 1: Timeline of the game

the Mafia is financed by the resources committed by the government to law enforcement earlier in the period, it suffers a small additional disutility $\epsilon > 0$ in the event of actual conflict.

After the outcome of any conflict is determined and benefits are divided, Firms choose whether or not to reelect the Government official, who is understood to compete against an otherwise identical challenger. This election completes period $t = 1$. The probability of reelection $\boldsymbol{\rho}$ can be represented by a finite-dimensional vector specifying the probability of reelection in each observationally distinct (for the Firms) situation. At $t = 2$, the sequence of decisions is the same as at $t = 1$, with the exception that there is no reelection decision at the end of the period. The timeline of the game is summarized in Figure 1.

3.1 Payoffs

The Mafia's utility is its net revenue (fees collected minus costs imposed in defeat). The Government's utility is any tax revenue not spent $((1 - \lambda^t)\tau)$ in each period. Each Firm's utility is the benefit it realizes from the contract (as enforced) net of taxes, minus the fee paid each period. The Government discounts future-period utility with discount factor δ. This can be interpreted either as impatience or as a measure of political instability or uncertainty. The higher δ is, the more certain the government is that the only reason it would not be in office next period would be electoral defeat. Other players are assumed not to discount the future because having them do so does not alter the analysis and adds notation.

4 Equilibria

Our solution concept is Perfect Bayesian Equilibrium. Additionally, we require that, when a player is capable of credibly committing to any of a set of behavioral strategies, she play a behavioral strategy that maximizes her *ex ante* expected utility at the earliest point in the history of the game at which such a credible commitment is possible. This requirement restricts our attention to those equilibria in which the Firms induce, via their choice of a reelection rule ($\boldsymbol{\rho}$), that behavior on the part of the Mafia and Government that the Firms prefer. Because Firms are indifferent between reelecting the Government official or not, such a choice of $\boldsymbol{\rho}$ is credible.

Our primary substantive interests concern what happens in $t = 1$. Period $t = 2$ is interesting substantively insofar as we can think of the government as being term-limited. We solve using backward induction.

4.1 Period 2

In the absence of a cost of challenging the Mafia, the Government is indifferent over all $\gamma^2 \in [0, 1]$. In the presence of cost $\epsilon > 0$, the Government will not challenge in the second period ($\gamma^{2*} = 0$). Further, because the Government can keep unspent revenue and because it does not face future elections, it has no incentive to invest in law enforcement ($\lambda^{2*} = 0$). The following lemma characterizes the effect of these choices on the Mafia's and Firms' actions in the second period:

Lemma 1 *In the second period, the Mafia always chooses fees such that both Firms accept its offers.*

Proof. See Appendix. ■

The intuition here is that in the second period the Firms and Mafia can ignore the government for the reasons discussed above. Consequently, the Mafia sets its fees to extract the greatest rents it can from the Firms. Since the Mafia can price discriminate, it turns out that the Mafia sets the highest possible fees such that both Firms are willing to hire the Mafia rather than appeal to the Government.

4.2 Period 1

4.2.1 Reelection

Firms' reelection decisions do not affect their future utilities, and so they are indifferent between reelecting the Government and not doing so. Although the Firms' reelection rule does not affect future payoffs, it does impact first-period payoffs by altering the first-period incentives of the Government and Mafia. Since, at the time of the reelection decision the Firms are indifferent, any reelection rule is credible. We focus, as discussed above, on reelection rules that maximize the Firms' expected utilities. In particular, we assume that Firms behave in a manner that induces "good" behavior on the part of Government in $t = 1$. There are four observably distinct situations under which the Firms must decide whether or not to reelect:

1. There is conflict between the Mafia and Government in $t = 1$ and the Mafia wins.

2. There is conflict between the Mafia and Government in $t = 1$ and the Government wins.

3. Both Firms hire the Mafia and there is no conflict.

4. Neither Firm hires the Mafia and there is no conflict.

Let M^t be the event that the Mafia wins in t and G^t be the event that the Government wins in t, given that there is conflict between the Mafia and Government in t. Let NG^t be the event that no conflict between the Mafia and Government occurs in t after neither Firm has hired the Mafia. Let NM^t be the event that no conflict between the Mafia and Government occurs in t after the Firms have hired the Mafia. The set $\{M^t, G^t, NM^t, NG^t\}$ can be thought of as the range of the outcome function, whose arguments include $\boldsymbol{\mu}^t, \gamma^t, \lambda^t$, and τ; to simplify notation, we suppress the functional representation below.

Let ρ_{M^1}, ρ_{G^1}, ρ_{NM}, and ρ_{NG} be the probabilities of reelection that correspond to each of the four outcomes enumerated above, respectively, where $\boldsymbol{\rho} = (\rho_{M^1}, \rho_{G^1}, \rho_{NM}, \rho_{NG}) \in [0, 1]^4$. Given that the Firms are indifferent over $\boldsymbol{\rho}$ at the time of the election, their optimal choice is induced by their preferences over the effects of that choice on other players' behavior. Accordingly, we return to the determination of the reelection rule after deriving the Mafia's and the Government's best responses to it.

4.2.2 Will the Government Challenge?

As noted earlier, the Government makes two choices: the level of investment into law enforcement (λ) and the probability of challenging the Mafia when both Firms hire the Mafia (γ). Somewhat surprisingly, the Firms' induced preferences over γ at the time of the Government's action are such that, once both Firms have paid the Mafia, they are expectationally indifferent between the Government challenging with certainty ($\gamma^1 = 1$) and the Government not challenging ($\gamma^1 = 0$). Two factors drive this fact. First, once both Firms hire the Mafia, the implemented contract is the same whether the Government challenges and wins, challenges and loses, or does not challenge at all. Second, because they are risk neutral, the Firms' *ex ante* expected utility from the lottery that corresponds to a first-period conflict equals their utility when there is no conflict. This does not, of course, mean that Firms are indifferent over the Government's strategy. Rather, Firms care only to the extent that Government behavior impacts the fees the Mafia charges. This intuition is formalized in the following lemma which is instrumental in solving for equilibrium behavior.

Lemma 2 *The Firms' preferences over Government action γ are completely induced by the effects of that action on the Mafia's choices.*

We can, thus, think of the Government as choosing the probability of challenging the Mafia (γ) to maximize the probability of reelection, given the level of investment in law enforcement (λ^1) and the reelection rule ($\boldsymbol{\rho}$). With this in mind, we can characterize the Government's best-response correspondence with respect to its choice of γ^1. The Government's expected utility from challenging with certainty is:

$$E[u_G^2(\gamma^1 = 1, \boldsymbol{\rho}^*, \lambda^1, \lambda^{2*}|\boldsymbol{\mu}^1 = (1,1))] = (1 - \Pr(M^1|\lambda^1, \tau))\rho_{G^1}\delta\tau + \Pr(M^1|\lambda^1, \tau)\rho_{M^1}\delta\tau.$$

The Government's expected utility from never challenging is:

$$E[u_G^2(\gamma^1 = 0, \boldsymbol{\rho}^*, \lambda^1, \lambda^{2*}|\boldsymbol{\mu}^1 = (1,1))] = \rho_{NM}\delta\tau.$$

Comparing these, we find that the Government's best response correspondence is

$$\gamma^{1*}(\lambda, \boldsymbol{\rho}; \cdot) = \begin{cases} 1 & \text{if } \rho_{M^1}\Pr(M^1|\lambda^1, tau) + \rho_{G^1}(1 - \Pr(M^1|\lambda^1, \tau)) > \rho_{NM} \\ \gamma' \in [0,1] & \text{if } \rho_{M^1}\Pr(M^1|\lambda^1, \tau) + \rho_{G^1}(1 - \Pr(M^1|\lambda^1, \tau)) = \rho_{NM} \\ 0 & \text{if } \rho_{M^1}\Pr(M^1|\lambda^1, \tau) + \rho_{G^1}(1 - \Pr(M^1|\lambda^1, \tau)) < \rho_{NM}. \end{cases} \quad (1)$$

From this condition it is clear that the Firms can always choose a ρ_{NM} such that the government will challenge.

A Benchmark

An instructive question, within this context, is whether it is important that the Government have the ability to challenge the Mafia even when neither Firm solicits the Government's help. What, for instance, would happen if the Government could only engage in conflict with the Mafia if appealed to by one of the Firms? This would be equivalent to restricting γ to be equal to 0. It turns out that if the Government does not have the authority to challenge the Mafia on its own, then regardless of tax policy or investment in law enforcement, the Mafia dominates the economy. The combination of the Firms' ability to appeal to the Government for protection against the Mafia and the arbitrarily stiff penalty that the Government can impose on the Mafia are never sufficient to induce the Firms to finance the Government if the Government cannot also be expected to challenge the Mafia when the Firms do not appeal to the Government directly.

Proposition 1 *If the government cannot challenge the Mafia unless appealed to by one of the Firms (that is, γ^t is restricted to be 0), the game has a unique equilibrium in which the Mafia completely replaces the Government as the provider of enforcement services in both periods, and the Firms choose not to fund the government at all ($\tau = 0$).*

Proof. See appendix. ■

As we will see in the remainder of the paper, allowing the Government to challenge the Mafia, unsolicited, radically alters players' equilibrium play.

4.2.3 Firms' Choice of Whether to Hire the Mafia

Consider now the Firms' (possibly mixed) choice of whether to hire the Mafia ($\boldsymbol{\mu}^1$). We use two facts in deriving the Firms expected utilities over their choice of whether to hire the Mafia. First, as demonstrated in Lemma 2, if both Firms hire the Mafia, their expected utility from the contract is the same whether or not the Government challenges the Mafia. Second, if both Firms hire the Mafia they get the same contract division that the Government would enforce. This second fact may seem like a strong symmetry assumption. To see why it is a reasonable reduced form, suppose that the Mafia favored one Firm or the other in its enforcement of the contract when hired by both Firms. In this case, the disadvantaged Firm would have an incentive to appeal to the Government rather than hire the Mafia. To counter-act this incentive, the Mafia would have to lower the fee it charged the disadvantaged Firm or lose that Firm as a customer and be forced into conflict with the Government. The Mafia's fee maximizing strategy, then, is to treat the two Firms equally if hired by both.

Thus, we can write F1's expected utility from hiring the Mafia with certainty as:

$$
\begin{aligned}
E[u_1^1(\mu_1^1 = 1, \mu_2^1)] = {} & \mu_2^1[\alpha^1(1-\tau) + \frac{1}{2}(1-\tau)(1-\Pr(M^2|N^1))] \\
& + (1-\mu_2^1)[\Pr(M^1|\lambda,\tau)(1-\tau+\frac{1}{2}(1-\tau)(1-\Pr(M^2|M^1))) \\
& + (1-\Pr(M^1|\lambda,\tau))(\alpha(1-\tau)+\frac{1}{2}(1-\tau)(1-\Pr(M^2|G^1)))] - \phi_1^1.
\end{aligned}
$$

F1's expected utility from appealing to the Government with certainty is:

$$
\begin{aligned}
E[u_1^1(\mu_1^1 = 0, \mu_2^1)] = {} & \mu_2^1[\Pr(M^1|\lambda,\tau)(0+\frac{1}{2}(1-\tau)(1-\Pr(M^2|M^1))) \\
& + (1-\Pr(M^1|\lambda,\tau))(\alpha(1-\tau)+\frac{1}{2}(1-\tau)(1-\Pr(M^2|G^1)))] \\
& + (1-\mu_2^1)[\alpha^1(1-\tau)+\frac{1}{2}(1-\tau)(1-\Pr(M^2|N^1)))].
\end{aligned}
$$

Similarly, we can write for F2:

$$
\begin{aligned}
E[u_2^1(\mu_1^1, \mu_2^1 = 1)] = {} & \mu_1^1(((1-\alpha^1)(1-\tau)+\frac{1}{2}(1-\tau)(1-\Pr(M^2|N^1))) \\
& + (1-\mu_1^1)[\Pr(M^1|\lambda,\tau)((1-\tau)+\frac{1}{2}(1-\tau)(1-\Pr(M^2|M^1))) \\
& + (1-\Pr(M^1|\lambda,\tau))((1-\alpha^1)(1-\tau)+\frac{1}{2}(1-\tau)(1-\Pr(M^2|G^1)))] - \phi_2^1,
\end{aligned}
$$

and

$$
\begin{aligned}
E[u_2^1(\mu_1^1, \mu_2^1 = 0)] = {} & \mu_1^1[\Pr(M^1|\lambda,\tau)(\frac{1}{2}(1-\tau)(1-\Pr(M^2|M^1))) \\
& + (1-\Pr(M^1|\lambda,\tau))((1-\alpha^1)(1-\tau)+\frac{1}{2}(1-\tau)(1-\Pr(M^2|G^1)))] \\
& + (1-\mu_1^1)((1-\alpha^1)(1-\tau)+\frac{1}{2}(1-\tau)(1-\Pr(M^2|N^1))).
\end{aligned}
$$

Notice that $\Pr(G^1)\Pr(M^2|G^1) + \Pr(M^1|\lambda,\tau)\Pr(M^2|M^1) = \Pr(M^2)$ and $\Pr(M^2|N^1) = \Pr(M^2)$. Making these substitutions and simplifying, we obtain that $\mu_2^1 = 1$ is the best response for F2 if

$$(1-\tau)\Pr(M^1|\lambda,\tau)[\mu_1^1(1-2\alpha^1)+\alpha^1) > \phi_2^1 \tag{2}$$

and $\mu_1^1 = 1$ is the best response for F1 if

$$(1-\tau)\Pr(M^1|\lambda,\tau)[\mu_2^1(2\alpha^1-1)+(1-\alpha^1)] > \phi_1^1. \tag{3}$$

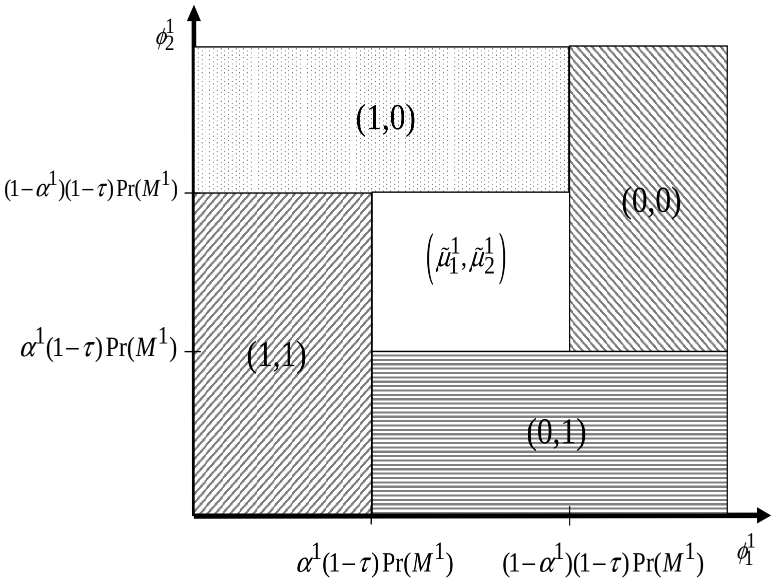

Figure 2: When the Firms Hire the Mafia as a Function of the Mafia's Fees.

Rearranging terms shows that F2 is indifferent if

$$\mu_1^1 = \frac{\phi_2^1}{(1-2\alpha^1)(1-\tau)\Pr(M^1|\lambda,\tau)} - \frac{\alpha^1}{(1-2\alpha^1)} \equiv \tilde{\mu}_1^1,$$

and that F1 is indifferent if

$$\mu_2^1 = -\frac{\phi_1^1}{(1-2\alpha^1)(1-\tau)\Pr(M^1|\lambda,\tau)} + \frac{(1-\alpha^1)}{(1-2\alpha^1)} \equiv \tilde{\mu}_2^1.$$

Figure 2 illustrates the actions taken by the two Firms in equilibrium for all possible fees charged by the Mafia in the first period ($\boldsymbol{\phi}^1$). If the fees charged to each Firm are sufficiently high, then neither Firm hires the Mafia (region $(0,0)$). Similarly, if the fees are sufficiently low, both Firms hire the Mafia (region $(1,1)$). The reason these two regions are not symmetric is that Firm 2 is the Firm disadvantaged in the contract. As a result, Firm 2 is more inclined to hire the Mafia and will do so for higher fees. Of course, since the Mafia can price discriminate, it can also set the fees such that only one Firm hires it or it can choose moderate fees for both Firms that induce a mixed strategy response.

4.2.4 The Mafia's Choice of Fees

Consider next the Mafia's choice of what fee to charge each Firm ($\boldsymbol{\phi}^1$). If the Mafia chooses $\boldsymbol{\phi}^1$ such that $\boldsymbol{\mu}^1 = (\tilde{\mu}_1^1, \tilde{\mu}_2^1)$, then its expected utility is

$$E[u_M(\boldsymbol{\phi}^1, \boldsymbol{\mu}^1 = (\tilde{\mu}_1^1, \tilde{\mu}_2^1); \mu^{2*}, \lambda^*, \gamma^*, \boldsymbol{\phi}^{2*}, \tau)] = \tilde{\mu}_1^1\phi_1^1 + \tilde{\mu}_2^1\phi_2^1 -$$
$$k(1 - \Pr(M^1|\lambda^{1*}, \tau)) \left[\tilde{\mu}_1^1(1 - \tilde{\mu}_2^1) + \tilde{\mu}_2^1(1 - \tilde{\mu}_1^1) + \gamma\tilde{\mu}_1^1\tilde{\mu}_2^1\right] + (1 - \tau)\Pr(M^2|\lambda^{2*}, \tau),$$

which is linear in ϕ_1^1 and in ϕ_2^1. This linearity implies that if the Mafia chooses fees that will induce the mixed strategy it will only consider fees that induce one of the four corners of the mixed strategy region in Figure 2. Since these four corners correspond to the pure strategy equilibria, we can restrict attention to the optimal fee choices that induce each of the four pure strategy combinations. Moreover, all fee choices that induce only one Firm to hire the Mafia are dominated by the optimal fee choice that induces both Firms to hire the Mafia. The reason for this is two-fold. First, the Mafia extracts higher total fees when it is hired by both Firms. Second, the Government and Mafia are certain to be in conflict if only one Firm hires the Mafia, whereas if both Firms hire the Mafia, then the Government and Mafia are in conflict only if the Government choose to challenge, which occurs with probability γ. Consequently, we can restrict attention to the optimal fee that induces both Firms to hire the Mafia and fees that induce neither Firm to hire the Mafia.

If the Mafia chooses $\boldsymbol{\phi}^1$ such that both Firms pay, i.e., $\boldsymbol{\mu}^1 = (1, 1)$, then it must prefer the highest possible fees such that they do. Hence, from (2) and (3), we get $\phi_1^1 = \alpha^1(1 - \tau)\Pr(M^1|\tau, \lambda^{1*})$ and $\phi_2^1 = (1 - \alpha^1)(1 - \tau)\Pr(M^1|\tau, \lambda^{1*})$, and so:

$$E[u_M(\boldsymbol{\phi}^1, \boldsymbol{\mu}^1 = (1, 1); \boldsymbol{\mu}^{2*}, \lambda^*, \gamma^*, \boldsymbol{\phi}^{2*}, \tau)] =$$
$$(1 - \tau)Pr(M^1|\lambda^{1*}, \tau) - k\gamma^*(1 - Pr(M^1|\lambda^{1*}, \tau)) + (1 - \tau)Pr(M^2|\lambda^{2*}, \tau) \quad (4)$$

If the Mafia chooses $\boldsymbol{\phi}^1$ such that neither Firm hires it ($\boldsymbol{\mu}^1 = (0, 0)$), then

$$E[u_M(\boldsymbol{\phi}^1, \boldsymbol{\mu}^1 = (0, 0); \boldsymbol{\mu}^{2*}, \lambda^*, \gamma^*, \boldsymbol{\phi}^{2*}, \tau)] = (1 - \tau)\Pr(M^2|\lambda^{2*}, \tau). \quad (5)$$

From these expected utilities we determine the optimal choice of fees by the Mafia. The Mafia will choose fees that induce the Firms not to hire it if the threat of Government challenge and censure is sufficiently large to more than offsets the benefit associated with collecting fees. Otherwise, the Mafia will choose fees that induce both Firms to hire it. Formally,

$$\boldsymbol{\phi}^1 = (\alpha^1(1-\tau)\Pr(M^1|\tau,\lambda^{1*}),\ (1-\alpha^1)(1-\tau)\Pr(M^1|\tau,\lambda^{1*})) \tag{6}$$

if $E[u_M(\boldsymbol{\phi}^1, \boldsymbol{\mu}^1 = (1,1);\cdot)] \geq E[u_M(\boldsymbol{\phi}^1, \boldsymbol{\mu}^1 = (0,0);\cdot)]$, which, from equations (4) and (5), yields

$$\gamma^* k < (1-\tau)\frac{\Pr(M^1|\lambda^{1*},\tau)}{1-\Pr(M^1|\lambda^{1*},\tau)}. \tag{7}$$

Otherwise, the Mafia is indifferent over any pair (ϕ_1^1, ϕ_2^1) such that neither Firm hires the Mafia.

4.2.5 The Government's Resource Allocation Decision

The Government chooses the amount of tax revenues to allocate to law enforcement (λ^1) such that

$$\lambda^1 \in \arg\max E[u_G^1(\lambda^1, \tau, \boldsymbol{\rho}^*(\tau,\cdot), \boldsymbol{\mu}^{1*}(\boldsymbol{\phi}^{1*}(\cdot),\cdot), \gamma^*(\boldsymbol{\rho}^*(\cdot), \lambda^1, \cdot)], \tag{8}$$

where

$$\begin{aligned} E[u_G^1(\lambda^1,\cdot)] &= (1-\lambda^1)\tau + [(1-\mu_1^{1*}(\cdot))(1-\mu_2^{1*}(\cdot))\rho_{NG}^*(\cdot) \\ &+ (\mu_1^{1*}(\cdot)(1-\mu_2^{1*}(\cdot)) + \mu_2^{1*}(\cdot)(1-\mu_1^{1*}(\cdot)))(\Pr(M^1|\lambda^1,\tau)\rho_{M^1}^*(\cdot) + (1-\Pr(M^1|\lambda^1,\tau)\rho_{G^1}^*(\cdot)) \\ &+ \mu_1^{1*}(\cdot)\mu_2^{1*}(\cdot)(\gamma^*(\cdot)(\Pr(M^1|\lambda^1,\tau)\rho_{M^1}^*(\cdot) + (1-\Pr(M^1|\lambda^1,\tau))\rho_{G^1}^*(\cdot)) + (1-\gamma^*(\cdot))\rho_{NM}^*(\cdot))]\delta\tau. \end{aligned} \tag{9}$$

In order to determine the possible equilibrium paths of play in all subgames beginning with the Government's choice of how much to invest in law enforcement (λ^1), recall that, in equilibrium, either both Firms hire the Mafia ($\boldsymbol{\mu}^1 = (1,1)$) or neither does ($\boldsymbol{\mu}^1 = (0,0)$). Thus, we can restrict attention to the equilibrium behavioral strategy profiles in which $\boldsymbol{\mu}^1 = (0,0)$ and $\boldsymbol{\mu}^1 = (1,1)$.

Notice, from equation (1) that the Government's choice of whether or not to challenge the Mafia (γ) is a function of the Government's resource allocation decision (λ). It turns out that the Government always chooses a resource allocation such that it will challenge the Mafia with probability 0 or 1. This is summarized in the following Lemma.

Lemma 3 *The Government will never choose* λ^1 *such that* $\gamma \in (0,1)$

Proof. See appendix. ■

This result implies that we can restrict attention to cases where, if both Firms hire the Mafia, the government never challenges ($\gamma = 0$) or challenges with certainty ($\gamma = 1$). Moreover, if the government never challenges in equilibrium, then it has no incentive to invest in law enforcement since it is never be called upon to fight. Thus, when considering the possibility that investment in law enforcement is positive ($\lambda^1 > 0$), we can restrict attention to cases where $\gamma = 1$, which, from equation (1) implies that $\rho_{NM} < Pr(M|\lambda^1, \tau)\rho_M + (1 - Pr(M|\lambda^1, \tau))\rho_G$.

Recall from equation (7) that the Mafia charges fees that lead both Firms to hire the Mafia only if $k < \frac{(1-\tau)}{\gamma} \frac{Pr(M^1|\lambda^1,\tau)}{1-Pr(M^1|\lambda^1,\tau)}$. Clearly, if the Mafia chooses fees such that neither Firm hires it, then the Government will again choose not to invest in law enforcement. Hence, in considering the possibility of positive λ^1 we can further restrict attention to cases where equation (7) is satisfied.

If the Government chooses a positive level of investment in law enforcement, it will make this choice to maximize its expected utility. Using the fact that $\gamma = 1$, this expected utility is given by:

$$E[u_G|\lambda^1, \boldsymbol{\mu}^1 = (1,1), \cdot] = (1-\lambda^1)\tau + \left[\left(Pr(M^1|\lambda^1,\tau)\rho_m + (1 - Pr(M^1|\lambda^1,\tau))\rho_G\right)\right]\delta\tau.$$

At an interior solution, the optimal level of investment in law enforcement, labeled λ', satisfies the following first-order condition:

$$-\frac{\partial Pr(M^1|\lambda', \tau)}{\partial \lambda} = \frac{1}{(\rho_G - \rho_M)\delta} \tag{10}$$

While this λ' is a local optimum, it may not be the Government's global optimum. We must also consider corner solutions. Clearly $\lambda = 1$ is never optimal, since the largest possible payoff in the future is $\delta\tau$, whereas by choosing $\lambda = 0$ the Government can insure itself a payoff of at least τ in this period. However, no investment in law enforcement ($\lambda^1 = 0$) could be optimal. N order to determine when λ' is preferred to $\lambda^1 = 0$, we need to consider two cases.

Case 1: $\rho_{NM} < Pr(M|0,\tau)\rho_M + (1 - Pr(M|0,\tau))\rho_G$.
In this case, if the Government chooses to deviate from $\lambda^1 = \lambda'$ to $\lambda^1 = 0$, γ nonetheless remains equal to 1. From the concavity of $-Pr(M|\lambda,\tau)$ and the definition of an optimum, it follows that $\lambda^1 = 0$ cannot be optimal in this case unless λ' is itself equal to 0. This occurs only if $-\frac{\partial Pr(M^1|\lambda',\tau)}{\partial \lambda} < \frac{1}{(\rho_G - \rho_M)\delta}$ for all $\lambda > 0$.

Case 2: $\rho_{NM} \in (Pr(M|0,\tau)\rho_M + (1 - Pr(M|0,\tau))\rho_G, Pr(M|\lambda',\tau)\rho_M + (1 - Pr(M|\lambda',\tau))\rho_G)$.
In this case, if the Government chooses to deviate from $\lambda^1 = \lambda'$ to $\lambda^1 = 0$, this will also lead it to switch from $\gamma = 1$ to $\gamma = 0$. Thus, we must compare:

$$E[u_G(\lambda',\gamma = 1)] = (1 - \lambda')\tau + (Pr(M|\lambda',\tau)\rho_M + (1 - Pr(M|\lambda',\tau))\rho_G)\,\delta\tau$$

to

$$E[u_G(0,\gamma = 0)] = \tau + (Pr(M|0,\tau)\rho_M + (1 - Pr(M|0,\tau))\rho_G)\,\delta\tau.$$

Comparing these two conditions, we find that the Government will choose $\lambda^1 = \lambda'$ in this case only if

$$[(\rho_G - \rho_M)\,(Pr(M|0,\tau) - Pr(M|\lambda',\tau))]\,\delta > \lambda', \tag{11}$$

otherwise it will choose $\lambda^1 = 0$.

We have established the conditions under which λ' is preferred to no investment in law enforcement, condition on the Firms hiring the Mafia. Now it reamins to consider the consistency of these conditions with the conditions under which the Firms hire the Mafia. There are three possiblities. From equation (7), if $k > (1-\tau)\frac{Pr(M^1|0,\tau)}{1-Pr(M^1|0,\tau)}$, then the Firms never hire the Mafia. If so, the Government never challenges, and so $\lambda^1 = 0$. If $k < (1-\tau)\frac{Pr(M^1|\lambda^1,\tau)}{1-Pr(M^1|\lambda^1,\tau)}$, then the Firms always hire the Mafia; the Government challenges and chooses $\lambda^1 = \lambda'$ if equation (11) is satisfied and the Government does not challenge and does not invest in law enforcement if it is not satisfied. Finally, we need to consider when $k \in \left((1-\tau)\frac{Pr(M^1|\lambda',\tau)}{1-Pr(M^1|\lambda',\tau)}, (1-\tau)\frac{Pr(M^1|0,\tau)}{1-Pr(M^1|0,\tau)}\right)$..

In this case, there is no pure strategy equilibrium. If the government chooses $\lambda^1 = \lambda'$, the Mafia will charge a fee that leads neither Firm to hire it. But then the Government's choice of λ^1 was not optimal, it should deviate to no investment in law enforcement. But, when it does so, the Mafia now wants to charge fees that lead both Firms to hire the Mafia, which again makes the Government's resource allocation decision sub-optimal. Thus, we look for mixed strategy equilibria.

Define $\hat{\lambda}$ as the choice of λ^1 such that

$$k = (1-\tau)\frac{Pr(M^1|\hat{\lambda},\tau)}{1-Pr(M^1|\hat{\lambda},\tau)}. \tag{12}$$

At this choice of investment in law enforcement, the Mafia is exactly indifferent between charging a fee that induces both Firms both to hire it and charging a fee that induces neither Firm to hire it. Let π be the probability that the Mafia choose $\boldsymbol{\phi}^1$ such that neither Firm hires it and $1-\pi$ be the probability that the Mafia chooses fees such that both Firms hire it. Then, in equilibrium the Mafia must choose this probability such that $\hat{\lambda}^1$ is optimal for the Government. The Government's expected utility is:

$$E[u_G(\lambda^1|\tau,\pi)] = (1-\lambda^1)\tau + \left(\pi\rho_{NG} + (1-\pi)(Pr(M|\lambda^1,\tau)\rho_M + (1-Pr(M|\lambda^1,\tau))\rho_G\right)\delta\tau.$$

The Mafia chooses π such that the following holds:

$$-(1-\pi)\frac{\partial Pr(M|\hat{\lambda},\tau)}{\partial\lambda} = \frac{1}{\delta(\rho_G-\rho_M)} \tag{13}$$

Combining all three cases, we have the following:

$$\lambda^{1*} = \begin{cases} 0 & \text{if } k > (1-\tau)\frac{Pr(M^1|0,\tau)}{1-Pr(M^1|0,\tau)} \text{ or} \\ & \quad k < (1-\tau)\frac{Pr(M^1|\lambda',\tau)}{1-Pr(M^1|\lambda',\tau)} \text{ and } \delta(\rho_G-\rho_M)\left[Pr(M^1|0,\tau) - Pr(M^1|\lambda',\tau)\right] < \lambda' \\ \lambda' & \text{if } k < (1-\tau)\frac{Pr(M^1|\lambda',\tau)}{1-Pr(M^1|\lambda',\tau)} \text{ and } \delta(\rho_G-\rho_M)\left[Pr(M^1|0,\tau) - Pr(M^1|\lambda',\tau)\right] \geq \lambda' \\ \hat{\lambda} & \text{if } k \in \left((1-\tau)\frac{Pr(M^1|\lambda',\tau)}{1-Pr(M^1|\lambda',\tau)}, (1-\tau)\frac{Pr(M^1|0,\tau)}{1-Pr(M^1|0,\tau)}\right), \end{cases} \tag{14}$$

where λ' is implicitly defined by equation (10) and $\hat{\lambda}$ is implicitly defined by equation (12).

4.2.6 The Firms' Reelection Rule

In light of these best responses, what reelection rule will the Firms adopt? Because any reelection rule is credible in equilibrium, we look for the reelection rule that maximizes the Firms' expected utilities given the paths of play described above. First, notice that ρ_{NM} affects the Government's choice of whether to challenge, but does not directly enter into the Government's choice of investment in law enforcement. In particular, the Government will challenge the Mafia only if $\rho_{NM} < \rho_{M^1} \Pr(M^1|\lambda^1,\tau) + \rho_{G^1}(1 - \Pr(M^1|\lambda^1,\tau))$. The Firms can always choose a ρ_{NM} that satisfies this constraint. Further note that the Firms choice of ρ_{NG} has no effect on any decisions and so any ρ_{NG} is optimal.

Finally, consider ρ_M and ρ_G. These two choices matter when we are in the case in which $k < \frac{Pr(M^1|\lambda',\tau)}{1-Pr(M^1|\lambda',\tau)}$. In this case, $\lambda^1 = \lambda'$ only if $\delta(\rho_G - \rho_M)\left[Pr(M^1|0,\tau) - Pr(M^1|\lambda',\tau)\right] \geq \lambda'$. Moreover, λ' is itself a function of ρ_G and ρ_M. Since the Firms want λ^1 to be as large as possible, increasing $\rho_G - \rho_M$ makes the Firms better off as long as it does not push them past the constraint so that λ^1 reverts from λ' to 0. The question thus arises of how increasing $\rho_G - \rho_M$ affects this constraint, since changing $\rho_G - \rho_M$ affects both the left- and right-hand sides of $\delta(\rho_G - \rho_M)\left[Pr(M^1|0,\tau) - Pr(M^1|\lambda',\tau)\right] \geq \lambda'$. It turns out that increasing $\rho_G - \rho_M$ makes this constraint more likely to be satisfied. To see this, differentiate both sides with respect to $\rho_G - \rho_M$. The derivative of the right-hand side is:

$$\frac{\partial \lambda'}{\partial(\rho_G - \rho_M)},$$

and the derivative of the left-hand side is:

$$\delta\left[Pr(M^1|0,\tau) - Pr(M^1|\lambda',\tau)\right] - \delta(\rho_G - \rho_M)\frac{\partial Pr(M^1|\lambda',\tau)}{\partial \lambda}\frac{\partial \lambda'}{\partial(\rho_G - \rho_M)}.$$

From equation (10) we know that $-\delta(\rho_G - \rho_M)\frac{\partial Pr(M^1|\lambda',\tau)}{\partial \lambda} = 1$. Substituting this in to the derivative of the left-hand side yields:

$$\delta\left[Pr(M^1|0,\tau) - Pr(M^1|\lambda',\tau)\right] + \frac{\partial \lambda'}{\partial(\rho_G - \rho_M)}.$$

Since $\delta\left[Pr(M^1|0,\tau) - Pr(M^1|\lambda',\tau)\right] > 0$, the derivative of the left-hand side is greater than the derivative of the right-hand side. Hency, as $\rho_G - \rho_M$ increases, it becomes more likely that the Government will choose a positive level of investment in law enforcement and that level of investment increases. Thus, the Firms want to choose $\rho_G - \rho_M$ as large as possible, which implies $\rho_G = 1$ and $\rho_M = 0$.

4.3 The Optimal Tax Rate

The only remaining action to be determined is the Firms' choice of the tax rate (τ). Since they set the tax rate before the realization of α^1 and the Firms' identities, their preferences over τ are unanimous. Given the best response correspondences $\boldsymbol{\rho}^*, \lambda^*, \boldsymbol{\phi}^*, \boldsymbol{\mu}^*$, and γ^*, we have:

$$E[u_F(\tau,\cdot)] = \begin{cases} \frac{(1-\tau)}{2}(2 - \Pr(M^2|0,\tau)) & \text{if } \tau \geq 1 - k\frac{1-\Pr(M^1|0,\tau)}{\Pr(M^1|0,\tau)} \\ (1-\tau)(1 - Pr(M^2|0,\tau) & \text{if } \tau < 1 - k\frac{1-Pr(M^1|\lambda',\tau)}{Pr(M^1|\lambda',\tau)} \text{ and} \\ & \delta\left[Pr(M^1|0,\tau) - Pr(M^1|\lambda',\tau)\right] < \lambda' \\ \frac{1}{2}(1-\tau)(2 - \Pr(M^1|\lambda',\tau) - \Pr(M^2|0,\tau)) & \text{if } \tau \leq 1 - k\frac{1-\Pr(M^1|\lambda',\tau)}{\Pr(M^1|\lambda',\tau)} \text{ and} \\ & \delta\left[Pr(M^1|0,\tau) - Pr(M^1|\lambda',\tau)\right] \geq \lambda' \\ \frac{1}{2}(1-\tau)\left(\pi + (1-\pi)\frac{1-\tau}{1-\tau+k} + 1 - Pr(M^1|0,\tau)\right) & \text{else.} \end{cases}$$

To find the optimal tax rate, one must maximize over each segment (realizing the each can occur in equilibrium) and then compare the value function at each local optimum to find the global optimum. However, it turns out to be difficult to identify conditions on primitives that give rise to each outcome without assuming further structure on the functional form of $\Pr(M|\lambda,\tau)$. It is more instructive to look at the dynamics of the relationship between Mafias, Governments, and Firms within each segment.

The essential trade-off when deciding over tax rates is driven by the Firms commitment problem. The Firms do not trust each other not to individually hire the Mafia in an attempt to extort money during their economic interactions. Taxation is beneficial to the Firms to

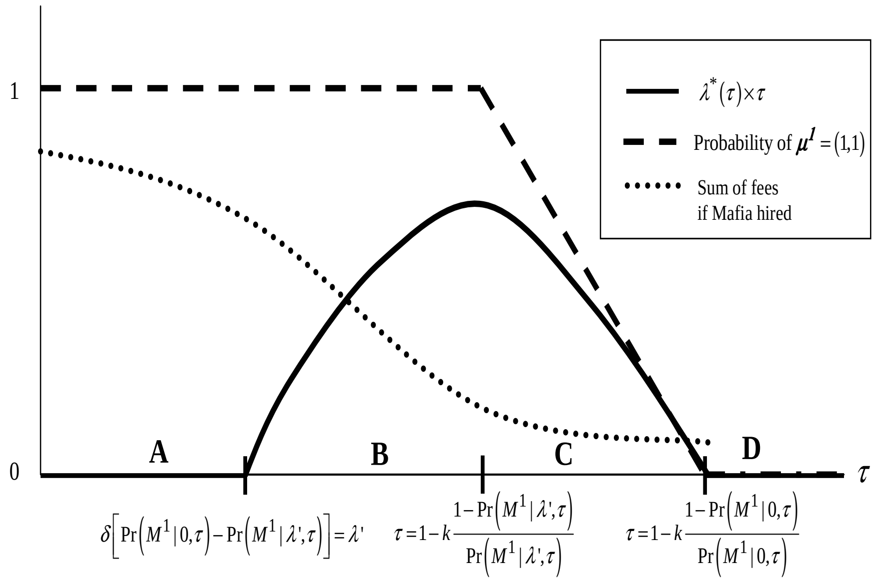

Figure 3: The Effect of Increasing Taxes on Law Enforcement Expenditures, the Frequency with Which the Mafia is Hired, and Fees Paid to the Mafia.

the extent that the revenues are direct toward law enforcement and thereby alleviate this commitment problem or limit the fees the Mafia is able to charge. Of course, providing such funds to the Government is costly to the Firms.

Figure 3 demonstrates how the dynamics of the model change as the tax rate increases. We consider each region of the x-axis (representing increasingly large tax rates and labeled A through D) of this figure in turn.

4.3.1 Region A: Unfunded Government and Dominant Mafia

In this region, the Firms are choosing a very low tax rate. As can be seen from the solid line in the figure (derived from equation (14)), when taxes are this low the Government invests nothing in law enforcement. The reason is that the marginal benefit of investing in law enforcement—increasing the probability of both defeating the Mafia and reaping electoral advantages—is very small when the total resources available are so low. As a consequence, the Government prefers to misappropriate the resources fully.

Very low taxes, then, give rise to a weak Government. A low tax rate also increases the value of economic transactions between the Firms. When the value of economic transactions is relatively large and the Government is weak, the Mafia is able to charge a large fee and still induce the Firms to hire it. The fact that the Firms always hire the Mafia in this region, is reflected in the dashed line in the figure. The effect of taxes on the fees charged is shown in the dotted line. The reason fees are high in this region is that when a lot of money is at stake and the Government is poorly positioned to defend revenues, each Firm worries a lot about being extorted by the other Firm and so is willing to pay more to the Mafia to protect itself. As taxes increase, even though law enforcement is not increasing ($\lambda = 0$ in this whole region), the Firms' willingness to pay decreases and so the fees charged decrease slightly. This can be seen in equation (6) evaluated at $\lambda^* = 0$.

The locally optimal level of taxation is easy to assess. Increasing taxes has two effects on the Firms expected utility. First, it decreases the amount of the value of economic transactions, which makes them worse off. Second, it decreases the fees that the Firms pay to the Mafia, which makes them better off. However, the negative effect grows at a faster rate than the positive effect as can be seen by taking the derivative of the expected utility with respect to taxes, for any value of τ in region A:

$$\frac{\partial E[u_F(\tau, \cdot)]}{\partial \tau} = -(1 - \Pr(M^2|0, \tau)) < 0.$$

Thus, the Firms prefer a tax rate of 0 to any other tax rate in this region, and rely entirely on the Mafia for enforcement since they cannot trust the Government not to expropriate taxes when Government funding is so modest.

The following Claims summarize the dynamics in region A.

Claim 1 *For all τ in A:*

1. *Governments invests 0 in law enforcement (i.e., $\lambda = 0$);*
2. *both firms hire the mafia and pay $\alpha'(1-\tau)\Pr(M|0)$ and $(1-\alpha')(1-\tau)\Pr(M|0)$, respectively;*
3. *the Government always challenges the Mafia;*

4. if the Government wins, Firms re-elect it; otherwise, the Government is replaced.

Claim 2 *In region A, increasing τ produces:*

1. *no change in $\lambda\tau$ and $\Pr(G)$;*

2. *no change in the probability that the firms hire the Mafia;*

3. *a decrease in the Mafia's absolute fees but no change in the proportion of firms' post-tax income paid to the Mafia.*

4.3.2 Region B: A Dominant Mafia Restrained by Government Law Enforcement

In this region, the Firms choose to fund the Government at a positive level, but nonetheless both Firms still hire the Mafia, as can be seen in the dashed line. What, then, is the advantage to the Firms of paying taxes?

When the Firms increase the tax rate into this range, the Government has an incentive to invest in law enforcement, as demonstrated in equation (11). The level of investment is λ', defined implicitly in equation (10). The reason the Government begins investing in law enforcement is that, as taxes increase, the marginal impact of law-enforcement spending on the probability of defeating the Mafia is large enough that the electoral incentive to spend now competes with the short-term desire to expropriate money. Indeed, as illustrated by the solid line, the level of spending on law enforcement is increasing in the tax rate. This can be seen analytically by noticing that the cross-partial of equation (10) is $-\delta\frac{\partial^2 Pr(M^1|\lambda',\tau)}{\partial\lambda\partial\tau} > 0$, which implies that λ' and τ are strategic complements.

Although the increased spending on law enforcement does not actually drive the Mafia out of business, the Firms still benefit from it. When law enforcement increases, the probability of the Mafia defeating the Government in a conflict decreases, which decreases the fees that the Mafia can charge the Firms, as derived in equation (6). Thus, as illustrated in the dotted line, in exchange for paying higher taxes, the Firms decrease the amount they must pay the Mafia for protection.

In order to determine the locally optimal level of taxation, contingent on being in this range, the Firms have to balance the positive effects of increased law enforcement and decreased fees against the negative effect of an increased tax burden. At an interior optimum, this locally optimal tax rate, labeled τ', is characterized by the following first-order condition:

$$2 - Pr(M|\lambda', \tau') - Pr(M|0, \tau') = -(1-\tau')\left(\frac{\partial Pr(M|\lambda', \tau')}{\partial \tau} + \frac{\partial Pr(M|\lambda', \tau')}{\partial \lambda}\frac{\partial \lambda'}{\partial \tau}\right),$$

where the right-hand side represents the marginal benefit in terms of decreased fees and the left-hand side represents the marginal cost in terms of increased taxes.

The following claims summarize the dynamics in region B.

Claim 3 *For all τ in B:*

1. *Governments invests λ' in law enforcement*;
2. *Both firms hire the mafia and pay $\alpha'(1-\tau)\Pr(M|0)$ and $(1-\alpha')(1-\tau)\Pr(M|0)$, respectively;*
3. *Government always challenges the Mafia;*
4. *If Government wins, Firms re-elect it; otherwise, the Government is replaced.*

Claim 4 *In region B, increasing τ produces:*

1. *an increase in $\lambda\tau$ and $\Pr(G)$;*
2. *no change in the probability that the firms hire the Mafia;*
3. *a decrease in the Mafia's absolute fees and in the proportion of the firms' post-tax income paid to the Mafia.*

4.3.3 Region C: The Government and Mafia Actively Compete.

As was demonstrated earlier, when the Firms increase the tax rate even further, there ceases to be a pure strategy equilibrium between the Government and the Mafia. If the Government invests tax revenues at a rate of λ', the Mafia will charge a fee that leads neither Firm to hire it. But if neither Firm hires the Mafia, then Government has no reason to invest in law enforcement. However, if the Government does not invest in law enforcement, the Mafia will set lower fees (since the threat of punishment is lower) and persuade the Firms to hire it. Thus, the Government and Mafia play a mixed strategy in this range.

As can be seen in equation (12), increasing the tax rate has two competing effects. First, as the tax rate increases, the Firms are less willing to pay fees to the Mafia in exchange for protection from extortion, which decreases the frequency with which the Firms hire the Mafia. Second, from the Government's perspective, investment in law enforcement is only productive when it is actually called on to challenge the Mafia (i.e., when both Firms hire the Mafia), since it is in these instances that such investment improves the Government's electoral chances. Hence, when the Firms decrease the frequency with which they hire the Mafia, the Government decrease its level of investment in law enforcement. Consequently, being hired is relatively less costly to the Mafia because they are less likely to lose a conflict with the Government, which allows the Mafia to charge lower fees. In equilibrium, these effects balance one another.

Thus, as taxes increase three things occur. First, as illustrated by the solid line, total expenditures on law enforcement decrease. Second, as illustrated by the dashed line, the frequency with which the Firms both hire the Mafia decreases. Finally, just as in the previous sections, as taxes increase the Firms become less willing to pay high fees because the threat of extortion is weaker. However, they decrease somewhat slower in this region because, unlike before, taxes are now also decreasing the level of law enforcement, which places upward pressure on the fees. Consequently, as shown by the dotted line, as taxes increase the fees the Mafia charges decrease, but more slowly than before.

Increasing taxes once again have positive and negative effects on the Firms' expected utilities. On the one hand, they decrease the fees paid and the probability of hiring the Mafia. On the other hand, they diminish the value of economic transactions. An interior locally optimal tax rate ($\hat{\tau}$), in this region, balances these effects and is characterized by the following first-order condition:

$$\frac{1}{2}\left[1+\left(\delta\frac{\partial Pr(M|\hat{\lambda},\hat{\tau})}{\partial\lambda}\right)^{-1}\frac{k}{1-\hat{\tau}+k}+1-Pr(M^1|0,\hat{\tau})\right]=$$

$$\frac{(1-\hat{\tau})}{2}\left[\delta\left(\frac{\partial^2 Pr(M|\hat{\lambda},\hat{\tau})}{\partial\lambda\partial\tau}+\frac{\partial^2 Pr(M|\hat{\lambda},\hat{\tau})}{\partial\lambda^2}\frac{\partial\hat{\lambda}}{\partial\tau}\right)\left(\delta\frac{\partial Pr(M|\hat{\lambda},\hat{\tau})}{\partial\lambda}\right)^{-2}\left(\frac{-k}{1-\hat{\tau}+k}\right)+\right.$$
$$\left.\left(\delta\frac{\partial Pr(M|\hat{\lambda},\hat{\tau})}{\partial\lambda}\right)^{-1}\left(\frac{k}{(1-\tau+k)^2}\right)\right],$$

where the left-hand side represents the marginal cost in terms of extra taxes paid and the right-hand side represents the marginal benefit in terms of decreased fees and decreased probability of hiring the Mafia.

The following claims summarize the dynamics in region C.

Claim 5 *For all τ in C:*

1. *Governments invests $\hat{\lambda}$ in law enforcement*;
2. *With positive probability, both firms hire the mafia and pay $\alpha'(1-\tau)\Pr(M|0)$ and $(1-\alpha')(1-\tau)\Pr(M|0)$, respectively. With the complementary probability, neither firm hires the Mafia;*
3. *If the firms hire the Mafia, Government challenges it;*
4. *If neither firm hires the Mafia, the firms re-elect Government with positive probability (but not certainty). If both firms hire the Mafia, the firms re-elect the Government if it defeats the Mafia, and replace it otherwise.*

Claim 6 *In region C, increasing τ produces:*

1. *a decrease in $\lambda\tau$ and $\Pr(G)$;*
2. *a decrease in the probability that the firms hire the Mafia;*
3. *a decrease in the Mafia's absolute fees but an increase in the proportion of firms' post-tax income paid to the Mafia.*

4.3.4 Region D: Dominant Government

If the Firms provide the Government with sufficient funding, then the Government can force the Mafia out of business in the first period, as occurs in the fourth segment of Figure 3. In this case, the Firms have turned over so many resources to the Government that they are only willing to pay a very low fee to the Mafia. The fees the Firms are willing to pay, however, are not large enough for the Mafia to be willing to accept the risk of punishment from the Government, even when the Government does not invest any extra resources in law enforcement. Thus, in this case, the Mafia prices itself out of the market, the Firms never hire the Mafia, and the Government expropriates all of the tax revenues. Clearly, in this situation, the Firms' expected utility is decreasing in the tax rate. Consequently, if the Firms choose this scenario, they will choose the lowest tax rate that does so: $\tau = 1 - k\frac{1-\Pr(M^1|0,\tau)}{\Pr(M^1|0,\tau)}$.

The following claims summarize the dynamics in region D.

Claim 7 *For all τ in D:*

1. *Governments invests 0 in law enforcement (i.e., $\lambda = 0$);*
2. *Neither firm hires the Mafia;*
3. *Government is re-elected with positive probability (but not certainty).*

Claim 8 *In region D, increasing τ produces no change in behavior on the path of play.*

5 Additional Results and Discussion

5.1 Taxation and Commitment

Economic agents in this model face a commitment problem. For any given level of taxation, it is Pareto inefficient for both Firms to hire the Mafia. This is because the enforced contract is the same whether they both hire the Mafia or both rely on the Government, but when they both hire the Mafia they also pay fees. The problem, of course, is that they do not trust each other not to individually hire the Mafia in an attempt to extort the entire value of the contract.

The Firms fund the government in order to solve this commitment problem. The threat of Government challenge, should both Firms hire the Mafia, makes it relatively less attractive to the Mafia to be hired. Consequently, government law enforcement and taxation can sometimes drive the Mafia out of the market or diminish the fees it can charge, thereby mitigating the commitment problem. Of course, weakening the Mafia comes at a price to the Firms—taxation. The Firms face a trade-off. They would like to create a situation where neither hires the Mafia. However, funding the government sufficiently to achieve this goal is costly. Consequently, even though appealing to the Mafia is *ex post* inefficient, the Firms will sometimes allow the Mafia to thrive. That is, there are scenarios where the Firms could drive the Mafia out of business by funding the government, but they choose not to do so because the increased tax burden would be more costly than the inefficiency of hiring the Mafia. This intuition is summarized in the following proposition.

Proposition 2 *For any tax rate the Firms prefer jointly not to hire the Mafia. Although the firms can always choose a tax rate,* $\tau = 1 - k\frac{1-Pr(M^1|0,\tau)}{Pr(M^1|0,\tau)}$*, that would lead them not to hire the Mafia, there are conditions under which they choose a lower tax rate that leads both of them to hire the Mafia.*

5.2 Taxation, Government Corruption, and Law Enforcement

The Firms fund the Government through taxation in order to increase law enforcement and, thereby, weaken the Mafia. The question arises, then, whether increasing government funding will actually lead to an increase in law enforcement, given the moral hazard problem that the Firms face vis-a-vis the government.

The government, in choosing how much to invest in law enforcement, balances two types of incentives. On the one hand, it is tempted to expropriate tax revenues. On the other hand, it has electoral incentives to invest in law enforcement. These electoral incentives come from the Firms' threat not to reelect the Government should it fail to challenge and defeat the Mafia. The Government, then, will act in an increasingly uncorrupt manner as the the electoral threat associated with losing to the Mafia increases relative to the appeal of expropriating tax revenues.

As is clear from the discussion above, and from Figure 3, the level of Government corruption is not monotonic in the level of funding. At very low and at very high levels of taxation the Government expropriates all of the tax revenue. At more moderate levels of taxation, corruption decreases as taxes increase as long as the marginal benefit associated with reelection continues to dominate the appeal of expropriation (region B). However, once taxes become high enough (region C), the level of corruption begins to increase with Government funding. This is because, as taxes increase in this range the Firms hire the Mafia less frequently. This weakens electoral incentives because conflict between the Government and Mafia (which is the source of the electoral incentives) becomes less frequent.

Proposition 3 *Government corruption is not monotonic in the level of taxation. In region A the government expropriates all tax revenues, in region B government corruption is decreasing in the tax rate, in region C government corruption is increasing in the tax rate, and in region D the government again expropriates all tax revenues.*

When Government corruption increases or decreases, it is not just the percentage of tax revenues that changes, but the absolute magnitude of resources invested in law enforcement. Thus, we have the following result.

Proposition 4 *The strength of Government law enforcement, measured as the probability that the Government defeats the Mafia when they are in conflict, is not monotonic in the level of taxation. It is flat in region A, increasing in region B, decreasing in region C, and flat again in region D.*

5.3 States and Mafias: Competing for Enforcement

An important question, within the context of this model, is under what conditions the Mafia will emerge dominant and under what conditions the Government will emerge dominant. We consider the effect of various parameters and choice variables on the relative prevalence of the Mafia and the Government.

5.3.1 The Effect of Taxation

The dashed line in Figure 3 shows that the frequency with which the Firms hire the Mafia is weakly decreasing in the tax rate. When taxes are very low, the Mafia dominates and the Government makes no effort to fight it. When taxes increase into region B, the Mafia continues to dominate the economy in the sense that neither Firm relies on the Government for protection. However, Government policy in this second region does have an effect on the Mafia. In particular, Government investment in law enforcement and the threat of punishment, decreases the fees that the Mafia charges the Firms. In this region, the Firms have used tax and electoral policy to successfully limit the strength, if not the ubiquity, of the Mafia.

As taxes increase even further, into region C, both the Mafia and Government are active in enforcing contracts. And, as taxes increase in this region, the frequency with which the Firms rely on the Government increases. Moreover, as taxes increase, the cut-point between regions B and C ($1 - k\frac{1-Pr(M^1|\lambda',\tau)}{Pr(M^1|\lambda',\tau)}$) shifts to the left, which reinforces the effect of taxes on the frequency of hiring the Mafia.

Finally, if taxes become high enough (region D), the Mafia is entirely eradicated. In order to achieve this outcome, the Firms must turn over enough money to the state in the form of taxes that the amount that the Mafia is able to charge in fees is no sufficient to overcome the risk of punishment that the Mafia faces when it provides revenue protection services.

The level of taxation that drives the Mafia out of business is $\bar{\tau} = 1 - k\frac{1-\Pr(M^1|0,\tau)}{\Pr(M^1|0,\tau)}$. Two comparative statics are evident. First, $\bar{\tau}$ is decreasing in the Government's natural advantage relative to the Mafia $(1 - \Pr(M^1|0, \tau)$. That is, in societies where extant Government institutions make the Government strong relative to Mafias, it is relatively inexpensive to drive the Mafia out of business. Second, $\bar{\tau}$ is decreasing in k. The larger the penalty the Government is able to impose on the Mafia, the easier it is to eradicate the Mafia. In the conclusion, we speculate briefly about possible extensions to the model that would allow us to endogenize and better interpret these parameters.

Proposition 5 *The frequency with which the Firms hire the Mafia is weakly decreasing in the tax rate. Moreover, if taxes are high enough, $\bar{\tau}$, the Mafia is entirely eradicated. The level of taxation necessary to eradicate the Mafia is decreasing in the Government's natural advantage relative to the Mafia ($1 - \Pr(M^1|0, \tau)$) and in the penalty the Government is able to impose on the Mafia (k).*

5.3.2 The Effect of Discounting

Earlier, we discussed an interpretation of the discount rate (δ) as a measure of political instability. When the Government is likely to be removed from office through some process other than elections, it will discount the future relatively heavily. The Government's discount rate impacts the frequency with which firms hire the Mafia in two ways. In region C the affect is direct; changes in the discount rate alter the Mafia's choice of how frequently to charge high versus low fees. Second, as can be seen in Figure 3, increasing the discount rate shifts the cutpoint between regions A and B and between regions B and C (by changing λ').

Consider, first, the impact of discounting in region C. In this region, the Firms hire the Mafia with probability $1-\pi$. Rearranging equation (13) shows that $\pi = 1 + \left(\frac{\partial Pr(M|\hat{\lambda},\tau)}{\partial \lambda}\delta\right)^{-1}$. It is clear from inspection that π is increasing in δ. That is, as discounting increases, the Firms rely on the Government with greater frequency. This is because, when the Government values the future more, it is willing to make a greater investment in law enforcement even though it has the opportunity to challenge the Mafia less frequently. Thus, in this region, the frequency with which the Mafia is hired is decreasing in the discount rate.

Now consider the effect of discounting on the likelihood of being in region A or B. As δ increases, the cutpoint between regions A and B shifts to the right. However, this has no impact on the frequency with which the Mafia is hired, since in both regions the Mafis is hired by both firms with certainty.

Finally, consider impact on the cutpoint between regions B and C. As is clear from equation (10), λ' is increasing in δ. The more the Government cares about the future, the more it is willing to invest in law enforcement in order to improve its chance of reelection. Hence, as the discount rate increases, the cutpoint between regions B and C shifts left, increasing the size of region C. Since, in region C the Firms rely on the Government with greater frequency than in region B, this effect on the cutpoint futher reinforces the finding that increased discounting decreases the frequency with which the Mafia is hired.

Proposition 6 *As δ increases, the frequency with which the Firms hire the Mafia weakly decreases.*

Building on the idea that the discount rate is a measure of instability, one interpretation of this proposition is that the Mafia is less likely to dominate the economy in societies with relatively stable political institutions.

5.3.3 The Effect of the Government's Underlying Capacity to Punish

There are two parameters of the model which measure the Government's underlying capacity to discipline the Mafia: $1 - Pr(M|0, \tau)$ and k. As discussed above, the former measures the Government's base strength relative to the Mafia without additional investment in law enforcement. The latter measures the level of punishment the Government can impose on the Mafia. It is clear from inspection that the cutpoints between regions B and C and between regions C and D are decreasing in both of these parameters. As these cutpoints shift left, the frequency with which the Mafia is hired decreases. Thus, the greater that Government's underlying capacity to discipline the Mafia, the less frequently the Mafia will be hired.

Proposition 7 *The frequency with which the Firms hire the Mafia is weakly decreasing in* $1 - Pr(M|0, \tau)$ *and in* k.

5.4 Inequality, Extortion, and Protection

As we have seen, on the equilibrium path either both Firms hire the Mafia or neither Firm hires the Mafia. However, it is instructive to notice that the two Firms have different motivations, out of equilibrium, underlying their behavior. Recall that Firm 1 is financially disadvantaged relative to Firm 2 ($\alpha < \frac{1}{2}$). As such, Firm 1 is tempted to hire the Mafia in order to extort Firm 2, since Firm 2 controls the bulk of economic resources. Firm 2, on the other hand, benefits greatly from economic transactions with Firm 1 and is, consequently less inclined toward extortion. Firm 2, then, is tempted to hire the Mafia not in order to extort Firm 1's resources but, rather, to provide itself with protection from extortion by Firm 1. This logic gives rise to the following proposition.

Proposition 8 *If* ϕ_i^1 *is in the interval* $(\alpha^1(1-\tau)Pr(M^1|\cdot), \alpha^1(1-\tau)Pr(M^1|\cdot))$*, the financially disadvantaged Firm (F1) is predatory, hiring the Mafia only if the other does not hire the Mafia (i.e., for the purpose of extorting the other Firm). In contrast, the financially advantaged Firm (F2) is defensive, hiring the Mafia only if the other Firm also hire the Mafia (i.e., for the purpose of protection from extortion).*

6 Conclusion

To be added.

7 Appendix

7.1 Proof of Lemma 1

Consider the Firms' choices. F1's expected utilities from accepting and from rejecting Mafia's offer are, respectively:

$$\begin{aligned} E[u_1^2(\mu_1^2 &= 1, \mu_2^2)] = \mu_2^2\alpha^2(1-\tau) + (1-\mu_2^2)\left[Pr(M^2|\cdot)(1-\tau) + (1-Pr(M^2|\cdot))\alpha^2(1-\tau)\right] - \phi_1^2 \\ E[u_1^2(\mu_1^2 &= 0, \mu_2^2)] = \mu_2^2\left[Pr(M^2|\cdot)\times 0 + (1-Pr(M^2|\cdot))\alpha^2(1-\tau)\right] + (1-\mu_2^2)\alpha^2(1-\tau). \end{aligned}$$

F1 prefers Mafia to Government if

$$\mu_2^2\alpha^2(1-\tau)Pr(M^2|\cdot) + (1-\mu_2^2))Pr(M^2|\cdot)(1-\tau)(1-\alpha^2) > \phi_1^2,$$

which is true only if

$$\mu_2^2 > \frac{1}{2\alpha^2-1}\left(\frac{\phi_1^2}{(1-\tau)Pr(M^2|\cdot)} - (1-\alpha^2)\right).$$

Similarly, for Firm 2:

$$\begin{aligned} &E[u_1^2(\mu_1^2, \mu_2^2 = 1)] = \\ &\quad \mu_1^2(1-\alpha^2)(1-\tau) + (1-\mu_1^2)\left[Pr(M^2|\cdot)(1-\tau) + (1-Pr(M^2|\cdot)(1-\alpha^2)(1-\tau)\right] - \phi_2^2 \end{aligned}$$

and

$$E[u_1^2(\mu_1^2, \mu_2^2 = 0)] = \mu_1^2\left[(1-Pr(M^2|\cdot))(1-\alpha^2)(1-\tau)\right] + (1-\mu_1^2)(1-\alpha^2)(1-\tau).$$

F2 prefers Mafia to Government if

$$\mu_1^2 > \frac{1}{1-2\alpha^2}\left(\frac{\phi_2^2}{(1-\tau)Pr(M^2|\cdot)} - \alpha^2\right).$$

Using these conditions, we have the following:

$$\boldsymbol{\mu}^2 = \begin{cases} (1,1) & \text{if } \phi_1^2 \leq \alpha^2(1-\tau)Pr(M^2|\cdot) \text{ and } \phi_2^2 \leq (1-\alpha^2)(1-\tau)Pr(M^2|\cdot) \\ (0,0) & \text{if } \phi_1^2 \geq (1-\alpha^2)(1-\tau)Pr(M^2|\cdot) \text{ and } \phi_2^2 \geq \alpha^2(1-\tau)Pr(M^2|\cdot) \\ (0,1) & \text{if } \phi_1^2 \geq \alpha^2(1-\tau)Pr(M^2|\cdot) \text{ and } \phi_2^2 \leq \alpha^2(1-\tau)Pr(M^2|\cdot) \\ (1,0) & \text{if } \phi_1^2 \leq (1-\alpha^2)(1-\tau)Pr(M^2|\cdot) \text{ and } \phi_2^2 \geq (1-\alpha^2)(1-\tau)Pr(M^2|\cdot) \end{cases}$$

Notice that, since the above conditions are exhaustive, there can be no mixed strategy equilibrium.

The Mafia's expected utility, as a function of the fees it chooses, is

$$E[u_M(\boldsymbol{\phi}^2|\boldsymbol{\mu}^2(\boldsymbol{\phi}^2))] = \mu_1^2\phi_1^2 + \mu_2^2\phi_2^2 - k(1 - Pr(M^2|\cdot)) \left[\mu_1^2(1-\mu_2^2) + \mu_2^2(1-\mu_1^2)\right] .$$

Clearly, the Mafia will never choose fees such that neither Firm hires it, since then the Mafia's payoff is 0. If it chooses a fee such that only Firm 2 hires it, it will choose the largest such fee $\phi_2^2 = \alpha^2(1-\tau)Pr(M^2|\cdot)$ in which case its expected utility is

$$\alpha^2(1-\tau)Pr(M^2|\cdot) - (1 - Pr(M^2|\cdot))k.$$

Similarly, if the Mafia chooses a fee such that only Firm 2 hires it, the Mafia's expected utility is:

$$(1-\alpha^2)(1-\tau)Pr(M^2|\cdot) - (1 - Pr(M^2|\cdot))k.$$

Finally, if the Mafia chooses fees such that both Firms hire it, it will choose the highest such fees, $\boldsymbol{\phi}^2 = (\alpha^2(1-\tau)Pr(M^2|\cdot), (1-\alpha^2)(1-\tau)Pr(M^2|\cdot))$, in which case the Mafia's expected utility is

$$(1-\tau)Pr(M^2|\cdot).$$

Clearly, this last is the highest of the expected utilities, so the Mafia chooses fees that induce both Firms to hire it in the second period. ■

7.2 Proof of Proposition 1

Because we solve by backward induction and because $\gamma^2 = 0$, the behavior in the second period of the modified game is like that of the unmodified game.

Expected utilities, in the modified game, are the same as in the unmodified game, so the Firms strategies with respect to hiring the Mafia are the same and so is the Mafia's expected utility, evaluated at $\gamma^1 = 0$. It is clear that equation (4) evaluated at $\gamma = 0$ is always greater than equation (5), so equation 6 describes the optimal choice of $\boldsymbol{\phi}^1$. On the equilibrium path, $\phi_1^1 > 0, \phi_2^1 > 0$, and both Firms hire the Mafia. Because the Government never fights the Mafia in equilibrium, $\lambda^{1*} = 0$, for all $\boldsymbol{\rho}^1$, which implies that $\tau^* = 0$. ■

7.3 Proof of Lemma 2

We compare the Firms' expected utilities associated with $\gamma^1 = 1$ and $\gamma^1 = 0$, respectively.

$$\begin{aligned}
E[u_{1,2}^2(\gamma^1 &= 1, \boldsymbol{\mu}^1 = (1,1), \cdot)] \\
&= \Pr(M^1|\lambda,\tau)\frac{1}{2}(1-\Pr(M^2|M^1))(1-\tau) + (1-\Pr(M^1|\lambda,\tau))\frac{1}{2}(1-\Pr(M^2|G^1))(1-\tau) \\
&= \frac{1}{2}(1-\tau)[1-\Pr(M^2|M^1)\Pr(M^1|\lambda,\tau) - \Pr(M^2|G^1)\Pr(G^1)] \\
&= \frac{1}{2}(1-\tau)(1-\Pr(M^2)) \\
&= E[u_{1,2}^2(\gamma^1 = 0, \boldsymbol{\mu}^1 = (1,1), \cdot)]
\end{aligned}$$

■

7.4 Proof of Lemma 3

There are two cases to consider:

1. $\rho_{NM} < Pr(M|0,\tau)\rho_M + (1 - Pr(M|0,\tau))\rho_G$

2. $\rho_{NM} \geq Pr(M|0,\tau)\rho_M + (1 - Pr(M|0,\tau))\rho_G$.

Since $Pr(M|\lambda,\tau)$ is decreasing in λ, we know from equation (1) that in case 1, $\gamma = 1$, regardless of λ^1.

Now consider case 2. From equation (1) we know that if $\lambda^1 = 0$, then $\gamma = 0$. This yields the following expected utility for the Government:

$$E[u_G(\lambda^1 = 0, \gamma = 0)] = \tau + \rho_{NM}\delta\tau$$

If, however, $\lambda^1 > 0$ and $\gamma \in (0,1)$, then the Government's expected utility is:

$$E[u_G(\lambda^1 > 0, \gamma \in (0,1))] = (1-\lambda^1)\tau + \gamma\left(Pr(M|0,\tau)\rho_M + (1-Pr(M|0,\tau))\rho_G\right)\delta\tau.$$

Note from equation (1) that if $\gamma \in (0,1)$, then $\rho_{NM} = Pr(M|0,\tau)\rho_M + (1 - Pr(M|0,\tau))$. Consider, then, the deviation from $(\lambda^1 > 0, \gamma \in (0,1))$ to $(\lambda^1 = 0, \gamma = 0)$. We have that:

$$E[u_G(\lambda^1 = 0, \gamma = 0)] = \tau + \rho_{NM}\delta\tau = \tau + \left(Pr(M|0,\tau)\rho_M + (1 - Pr(M|0,\tau))\right)\delta\tau,$$

which is clearly larger than $E[u_G(\lambda^1 > 0, \gamma \in (0,1))]$. Hence, if $\lambda^1 > 0$, then $\gamma \notin (0,1)$. Moreover, if $\gamma = 0$, then λ^1 must be 0. Thus , if $\lambda > 0$, then $\gamma = 1$. ■

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