Information Acquisition and Welfare*

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Abstract

We study information acquisition in a flexible framework with strategic complementarity or substitutability in actions and a rich set of externalities that are responsible for possible wedges between the equilibrium and the efficient acquisition of information. First, we relate the (in)efficiency in the acquisition of information to the (in)efficiency in the use of information and explain why efficiency in the use does not guarantee efficiency in the acquisition. Next, we show how the acquisition of private information affects the social value of public information (i.e., the comparative statics of equilibrium welfare with respect to the quality of public information). Finally, we illustrate the implications of our results in a few applications that include beauty contests, monetary economies with price-setting complementarities, and economies with negative production externalities.

Keywords: endogenous information, strategic complementarity/substitutability, externalities, efficiency, welfare

JEL classification: C72, D62, D83, E50

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1 Introduction

Many economic environments feature a large group of agents taking decisions under disperse information about relevant economic fundamentals affecting individual preferences and/or the profitability of investment opportunities.

In such environments, the information that agents optimally choose to collect about the underlying fundamentals is determined by their desire to align their actions with the fundamentals as well as with other agents' actions. Furthermore, because acquiring information is costly, the amount of information collected at the private level depends on the quality of public information provided by policy makers, statistics bureaus, and the like.

In this paper, we investigate how the amount of private information collected in equilibrium differs from the socially optimal one and relate the discrepancy between the two to the primitives of the environment, as well as to the way information is used in equilibrium. We then use such a characterization to revisit the social value of public information, shedding new light on how the desirability of transparency in public disclosures is affected by the endogenous response in the acquisition of private information.

To abstract from specific institutional details and identify general principles while retaining tractability, we conduct our analysis within the flexible quadratic-Gaussian family of economies studied in Angeletos and Pavan (2007). This framework allows for both strategic complementarity and substitutability in actions as well as for a rich set of externalities that create possible wedges between the equilibrium and the efficient use of information.

Contrary to Angeletos and Pavan (2007), however, we allow agents to choose the amount of private information they collect. We assume that the acquisition of private information is costly and allow for an arbitrary cost function. This permits us to interpret the cost of information acquisition either as the monetary cost of purchasing different sources of information or as the cognitive cost of processing available information, as in the rational inattention literature (e.g., Sims, 2003, and Maćkowiak and Wiederholt, 2009). Importantly, and realistically, we allow agents to change the amount of private information they collect in equilibrium in response to variations in the quality of information provided by policy makers, statistics bureaus, and other sources of public information. Recognizing the endogeneity of private information turns out to have major implications for the social value of public information as we explain below.

Our first result characterizes the amount of private information collected in equilibrium and establishes that the latter is decreasing in the precision of public information, with a degree of substitutability between the two that is increasing in the importance that agents assign to aligning their actions with those of other agents, that is, in the equilibrium degree of coordination.[1] Im-

[1] Although established in a different setting, the substitutability between public and private information is also documented in a recent paper by Myatt and Wallace (2012). The focus of that paper is, however, very different than the one in the present paper, as discussed below in the section on related literature.

portantly, we show that, while the intensity of the substitution effect depends on the strength of the coordination motive, its sign does not: irrespective of whether the economy features strategic complementarity or substitutability in actions, an increase in the precision of public information always leads to a reduction in the amount of private information collected in equilibrium.[2]

We then proceed by characterizing the amount of private information that a benevolent planner would like the agents to collect so as to maximize welfare, defined as the ex-ante expected utility of a representative agent. Such a characterization has no precedents in the literature and is one of the distinctive contributions of this work.

Perhaps surprisingly, we show that the amount of private information collected in equilibrium is typically inefficient, even in those economies where the use of information (that is, the mapping from information to actions) is efficient. The reason why efficiency in the use does not guarantee efficiency in the acquisition of information is that agents may suffer (or benefit) from the dispersion of individual actions in the cross section of the population (as, for example, in beauty contests, or in economies with price-setting complementarities). When such externality from dispersion has a direct, non-strategic, effect on individual utilities, it creates a wedge between the equilibrium and the efficient acquisition of private information, despite the economy responding efficiently to the information it collects. More precisely, the amount of private information collected in equilibrium is inefficiently low in those economies where agents benefit from the dispersion of individual actions in the population, whereas it is inefficiently high in those economies featuring a negative externality from dispersion.

Next, we consider economies where the inefficiency in the acquisition of information originates in the inefficiency in the use of information. We show that the amount of private information acquired in equilibrium is inefficiently low when the equilibrium degree of coordination exceeds the socially optimal one (that is, when the value that agents assign to aligning their actions is higher than the value that the planner would like them to perceive for them to process information efficiently). The opposite is true in economies where the equilibrium degree of coordination falls short of the socially optimal one: in this case, the amount of private information collected in equilibrium is too high compared to the socially efficient level. From Angeletos and Pavan (2007), we know that the discrepancy between the equilibrium and the socially optimal degrees of coordination is what drives the discrepancy between the equilibrium and the efficient sensitivity of individual actions to the different sources of information. Combining the results in this paper with those in Angeletos and Pavan (2007), we can then show that economies where agents overreact to public sources of information are also economies where agents underinvest in the acquisition of private information, whereas the opposite is true for economies where agents under-respond to public infor-

[2] The strength of the substitution effect between public and private information also depends on the elasticity of the function determining the cost of private information acquisition. In the paper, we compute bounds for the substitution effect across all possible cost functions, which we then use to sign the social value of public information in various cases of interest.

mation.[3] These results hold irrespective of whether the economy features strategic complementarity or substitutability in actions and irrespective of the cost of information acquisition.

Lastly, we consider economies where the inefficiency in the collection and use of information originates in the inefficiency of the complete-information allocation (that is, in the discrepancy between the complete-information allocation and the first-best allocation). We show that, in these economies, the amount of private information acquired in equilibrium is inefficiently low (respectively, high) if the sensitivity of the complete-information allocation to the fundamentals is inefficiently low (respectively, high). In other words, the direction in which society fails to respond adequately to changes in fundamentals under complete information determines the direction in which it fails to acquire the efficient amount of private information.

Clearly, the cases considered above do not cover all possibilities. There are economies where the inefficiency in the acquisition of information comes from a combination of the three sources identified above: (i) the discrepancy between the private and the social value of reducing the cross-sectional dispersion of individual actions, (ii) the inefficiency in the equilibrium use of information, (ii) the inefficiency of the complete-information allocation. By isolating the source of the inefficiency, the economies discussed above represent useful benchmarks that one can then use to examine more complex economies.

Importantly, the analysis of the inefficiency in the acquisition of private information has implications for the social value of public information. While previous research focused on the *partial* effect that more precise public information has on welfare holding constant the precision of private information, here we investigate its *total* effect, taking into account that private agents are bound to change their acquisition of private information in response to variations in the quality of available public information. In particular, we show that, irrespective of whether the economy features strategic complementarity or substitutability, recognizing the endogeneity of private information leads to a higher social value of public information if the amount of private information collected in equilibrium is inefficiently high, while the opposite is true if it is inefficiently low.

This last result follows directly from the crowding-out effect that more precise public information exerts on the acquisition of private information. When the economy collects too much private information, such crowding-out effect contributes positively to the social value of public information, whereas the opposite is true when the amount of private information collected in equilibrium falls short of the efficient level. Interestingly, recognizing the endogeneity of private information (namely, the fact that agents are bound to change the amount of private information they collect in equilibrium in response to variations in the quality of available public information) may change the sign of the social value of public information. More precisely, we show that there exists a critical threshold $\Delta > 0$ for the discrepancy between the equilibrium (α) and the socially optimal (α^*) degrees of coordination such that, whenever $\alpha - \alpha^* < \Delta$, acknowledging the endogeneity of private

[3] Note that this result does not follow from previous research: an excessively high sensitivity of equilibrium actions to public sources of information does not imply underinvestment in the equilibrium collection of private information.

information can turn the social value of public information from negative to positive, but never from positive to negative. Note that this result is true irrespective of whether or not the economy is efficient under complete information, of whether or not agents benefit from the dispersion of individual actions in the population, and irrespective of the details of the cost function for the acquisition of private information. Thus, whenever the importance that society assigns to aligning individual decisions does not exceed by too much the socially optimal level, the crowding-out effect that public information exerts on private information is never strong enough to turn negative the social value of public information.

In the last part of the paper, we apply our results to a few applications of interest. We start with a beauty contest framework analogous to the one studied in Morris and Shin (2002). We show that, recognizing the endogeneity of private information may change the sign of the social value of public information. This occurs when the amount of private information collected in equilibrium is inefficiently high. By inducing agents to reduce the amount of private information collected in equilibrium, more precise public information can thus contribute positively to welfare under the same parameters' configurations that were shown to lead to a negative social value of public information when ignoring the endogenous response in the acquisition of private information. Quite interestingly, we also show that the reverse is never possible. Even if, in some cases, recognizing the endogeneity of private information may reduce the social value of public information (this occurs when the amount of private information collected in equilibrium is inefficiently low), the latter remains positive whenever it was shown to be positive ignoring the crowding-out effect of public information on private information.

The second application is a monetary economy with monopolistic price competition, along the lines of those studied in Hellwig (2005), and Roca (2010). Contrary to the 'beauty-contests', these economies are characterized by the presence of a negative (non-strategic) externality from dispersion and by an inefficiently low degree of equilibrium coordination. Previous research has shown that, in these economies, more precise public information always contributes positively to welfare. Our contribution is in showing that the same conclusion holds when acknowledging the crowding out effect of public information on the acquisition of private information. This effect can reduce the social value of public information in those situations where the amount of private information collected in equilibrium is inefficiently low. However, it is never strong enough to overturn the direct positive effect that public information exerts on welfare by helping firms better align their pricing decisions.

The last application is a competitive consumer-producer economy with negative externalities from aggregate production, such as those originating from pollution. Contrary to the other two applications, in this economy agents' production decisions are strategic substitutes and there are no direct externalities from the dispersion of individual actions in the population. Moreover, because individual producers ignore the contribution of their production choices to the aggregate externality, this economy is characterized by an inefficiently high degree of coordination (that is, the level of strategic substitutability that the planner would like the agents to perceive is larger than the equilibrium one). Finally, contrary to the other two applications, this economy is inefficient not only in its collection and use of information but also in its functioning under complete information: production decisions overreact to changes in fundamentals relative to what is efficient. We then show that, in these economies, recognizing the crowding-out effects of public information on private information always increases the social value of public information, and can turn the latter positive in situations where it would have been negative ignoring the endogenous response in the acquisition of private information.

The rest of the paper is organized as follows. We briefly review the pertinent literature in Section 2. Section 3 presents the model. Section 4 characterizes the equilibrium collection of private information. Section 5 characterizes the efficient collection of private information. Section 6 studies the implications of the acquisition of private information for the social value of public information. Section 7 contains the applications described above. Section 8 concludes with a few final remarks. Finally, Appendix A contains proofs omitted in the main text, while Appendix B shows how the monetary economy with price-setting complementarities examined in Section 7 can be traced back to the abstract framework of this paper after appropriate linear-quadratic approximations.

2 Related literature

Social value of public information. This paper belongs to the literature that investigates the welfare implications of public information provision. In a highly debated article, Morris and Shin (2002) show that public information may have a detrimental effect on welfare in economies resembling Keynes' beauty contests. Because the strategic complementarity that agents perceive in their actions is not warranted from a social perspective, and because public information is more effective in aligning individual actions than private information, agents rely too much on public sources of information relative to what is efficient. Furthermore, because the agents' reliance on public information in turn increases with its precision, more precise public information can be welfare decreasing. Our paper contributes to this literature by showing that, because in these economies agents may collect too much private information relative to what is efficient, more precise public information, by inducing agents to cut on their collection of private information, may have a net positive effect on welfare in situations in which it was shown to have a negative effect ignoring the acquisition of private information.

Following up on Morris and Shin (2002), Cornand and Heinemann (2008), again in a beauty contest framework, show that more precise public information may have a positive effect on welfare when it reaches only a fraction of market participants.[4] Morris and Shin (2007) also consider an

[4]Morris and Shin (2002) also stimulated a debate on the empirical plausibility of their result, questioned by Svensson (2006) and reaffirmed by Morris, Shin and Tong (2006).

economy characterized by a 'semi-public' signal reaching only a fraction of agents in addition to the usual fully public signal, and show that, in the absence of direct externalities from dispersion, the fragmentation of information always leads to welfare losses. The welfare implications of both papers rest on the fact that some relevant information reaches only a share of market participants. In contrast, in our framework, the welfare effects of public information depend on the strategic substitutability between private and public information.

The analysis of the social value of information has been extended by Angeletos and Pavan (2007) to a large class of quadratic-Gaussian economies featuring both strategic complementarity and substitutability and a rich set of externalities. The analysis in the current paper is within the same framework as in that paper. As mentioned already, the key difference is that, in the present paper, we investigate the process by which agents respond to variations in the quality of public information by changing the amount of private information that they collect in equilibrium. We identify sources of inefficiency in this process and show how recognizing this process may contribute a different evaluation of the social value of public information.

Related is also Myatt and Wallace (2008), who consider a Lucas-Phelps island economy with several (imperfectly correlated) information sources, each of which is neither perfectly private nor perfectly public. They show that it is never optimal for a benevolent planner to provide a perfectly public or perfectly private signal. The welfare implications of more transparency in public information is also the theme of a few recent contributions to the macroeconomics literature on monetary policy with monopolistic competition and dispersed information (e.g., Hellwig, 2005, Lorenzoni, 2010, Roca, 2010, Angeletos, Iovino, and La'O, 2011).

None of the papers cited above looks at the interplay between the social value of public information and the inefficiency of the acquisition of private information, which is the distinctive feature of this paper.

Endogenous private information acquisition. The role of private information acquisition in coordination settings has been recently explored in Hellwig and Veldkamp (2009) and Myatt and Wallace (2012). Hellwig and Veldkamp (2009) show that strategic complementarities in actions induce strategic complementarity in private information acquisition (i.e., "agents who want to do what others do, want to know what others know"), which in turn may lead to multiple equilibria. A similar point is made in Maćkowiak and Wiederholt (2009); in a framework with rational inattention, they show that strategic complementarity in price-setting decisions leads to strategic complementarity in the price setters' allocation of attention. Myatt and Wallace (2012) consider endogenous information acquisition in a beauty-contest framework where agents have access to a variety of information sources. In their framework, the precision of each signal depends on both the clarity of the signal (the 'sender' noise, in their terminology) and the attention that an agent devotes to the signal (the 'receiver' noise). As in our model, allocating more attention to a signal entails a larger cost. When agents pay careful attention to the same source of information, the correlation in their signals endogenously increases, which in turn implies an increase in the publicity associated to the signal.[5]

Related is also a recent independent paper by Llosa and Venkateswaran (2012). That paper considers endogenous information acquisition in three applications of interest: a beauty contest similar to Morris and Shin (2002), a business cycle model in the spirit of Angeletos and La'O (2010), and a monetary economy as in Hellwig (2005). As in the current paper, efficiency in the use of information is shown to be no guarantee of efficiency in the collection of private information. However, contrary to the present paper, that paper does not identify general conditions (in terms of discrepancy between the private and social value of reducing dispersion, as well as between the equilibrium and the efficient use of information) that are responsible for inefficiencies in the acquisition process.

Importantly, none of the papers cited above looks at the social value of public information and at how the latter depends on the inefficiency of the acquisition of private information, which is one of the distinctive themes of the present paper.

A crucial feature of our model is that private agents respond to variations in the quality of available public information by changing the amount of private information they collect in equilibrium. A similar timing has been considered by Dewan and Myatt (2008, 2009). In a model of political leadership, party activists decide how much attention to pay to different leaders in order to coordinate their actions. Taking this into account, party leaders may decide to obfuscate the clarity of their communications. The focus of these papers is however very different from ours: party leaders maximize the probability of winning the election in these papers, whereas the planner maximizes welfare in our environment.

Endogenous information acquisition is also the theme of Demerzis and Hoeberichts (2007) and Wong (2008). Differently from us, these two papers assume that agents must choose the quality of their private information before observing the quality of public information, thus abstracting from the crowding-out effects that we document in this paper.

The same theme is also the focus of Colombo and Femminis (2008). That paper considers the same timing of private information acquisition as in the present paper. However, it restricts attention to a simple beauty-contest environment *à la* Morris and Shin (2002) and assumes a linear cost function for both private and public information thus leading to the prediction that, in equilibrium, only one type of information (either public or private) is collected. In addition to the difference in the generality of the framework, the scope of the two papers is fundamentally different. While that paper shows that allowing for an endogenous response in the collection of private information may increase the social value of public information, the current paper characterizes the

[5] Differently from Hellwig and Veldkamp (2009) where the attention devoted to each signal is binary (that is, agents choose whether or not to purchase a signal of given quality), Myatt and Wallace (2012) allow the attention allocated to the various signals to be a continuous variable. This difference turns out to have important implications for the equilibrium determinacy: while there are multiple equilibria in Hellwig and Veldkamp (2009), the equilibrium is unique in Myatt and Wallace (2012).

sources of inefficiency in the acquisition of private information and use the latter to reassess the social value of public information.

Crowding-out effects of public information. Our paper is also related to the literature that documents the crowding-out effects of more precise *exogenous public* information on the *endogenous public* information aggregated by prices and other public statistics (e.g., Vives (1993), Morris and Shin (2005), Amato and Shin (2006) and, more recently, Amador and Weill (2010), and Vives (2011)). In these papers, more precise exogenous public information, by inducing agents to rely less on their exogenous private information, has the perverse effect of reducing the informativeness of endogenous public signals. In contrast, in the present paper, more precise public information exerts a crowding-out effect on the agents' collection of private information. Whether such a crowding-out effect contributes positively or negatively to the social value of public information is then shown to depend on whether agents underinvest or overinvest in their acquisition of private information. In the paper, we identify primitive conditions for each case.

3 The environment

Agents and Information. We study a two-period economy populated by a continuum of agents of measure one, indexed by i and uniformly distributed over $[0,1]$. Each agent i observes noisy private and public signals about an underlying fundamental θ. In period -1, every agent knows the state of the economy θ_{-1}, which represents the common *ex-ante* expectation of the state variable θ in period 0. The fundamental evolves according to the stochastic process

$$\theta = \theta_{-1} + \varphi.$$

The shock φ, occurring at the beginning of period 0, is normally distributed with mean zero, variance σ_θ^2, and precision $p_\theta \equiv \sigma_\theta^{-2}$. After the realization of the shock, every agent i receives a public signal

$$y = \theta + \varepsilon$$

and a private signal

$$x_i = \theta + \xi_i,$$

where ε is normally distributed, independent of θ, with mean zero and precision p_y, and where each noise terms ξ_i is normally distributed, independent of θ, ε, and ξ_j $(j \neq i)$, with mean zero and precision p_{x_i}. While y is common knowledge among the agents, x_i is idiosyncratic to agent i and not observed by any of the other agents. The precision of the private signal may vary across agents and is determined endogenously by the amount of private information collected by the agent, as we explain in more details below.

The common posterior about θ given public information y is normally distributed with mean $\mathbb{E}\left[\tilde{\theta}\middle| y\right] = \frac{p_\theta \theta_{-1} + p_y y}{p_\theta + p_y}$ and precision $p\left[\tilde{\theta}\middle| y\right] = p_\theta + p_y$. We define $z \equiv \mathbb{E}\left[\tilde{\theta}\middle| y\right]$ and $p_z \equiv p_\theta +$

p_y. Private posteriors are normally distributed with mean $\mathbb{E}\left[\tilde{\theta}\middle| y, x_i\right] = \frac{p_z z + p_{x_i} x_i}{p_z + p_{x_i}}$ and precision $p\left[\tilde{\theta}\middle| y, x_i\right] = p_z + p_{x_i}$. Consistently with Angeletos and Pavan (2007), we refer to $p\left[\tilde{\theta}\middle| y, x_i\right]$ as to the 'accuracy' of agent i's information. Letting $\delta_i \equiv \frac{p_z}{p_z + p_{x_i}}$ be the weight of the public signal in the Bayesian projection of θ on (x_i, y), we then have that the posterior mean for each agent i is given by $\mathbb{E}\left[\tilde{\theta}\middle| y, x_i\right] = \delta_i z + (1 - \delta_i) x_i$.

Actions and Payoffs. Let $k_i \in \mathbb{R}$ denote agent i's action, $K \equiv \int_j k_j dj$ the mean action in the cross section of the population, and $\sigma_k^2 \equiv \int_j [k_j - K]^2 dj$ the dispersion of individual actions in the population. Each agent's preferences are characterized by the (expectation of) the Bernoulli utility function

$$U(k_i, K, \sigma_k, \theta).$$

As is standard in the literature, we assume that U is approximated by a second-order polynomial and that dispersion σ_k has only a second-order non-strategic external effect, so that $U_{k\sigma} = U_{K\sigma} = U_{\theta\sigma} = 0$ and that $U_\sigma(k, K, 0, \theta) = 0$, for all (k, K, θ).[6] The quadratic specification of the utility function ensures the linearity of the agents' best responses and of the efficient allocations.

In addition to the above conditions, we assume that partial derivatives satisfy the following conditions: (i) $U_{kk} < 0$, (ii) $\alpha \equiv -U_{kK}/U_{kk} < 1$, (iii) $U_{kk} + 2U_{kK} + U_{KK} < 0$, (iv) $U_{kk} + U_{\sigma\sigma} < 0$ and (v) $U_{k\theta} \neq 0$.

Condition (i) imposes concavity at the individual level, so that best responses are well defined, while Condition (ii) implies that the slope of best responses is less than one, which in turn guarantees uniqueness of the equilibrium. Conditions (iii) and (iv) guarantee that the first-best allocation is unique and bounded, and ensure concavity at the aggregate level. Finally, Condition (v) ensures that the fundamental θ affects equilibrium behavior.

Timing and Information Acquisition. The economy described above is the same as in Angeletos and Pavan (2007). To that economy we add an initial stage where agents choose the quality of their private information in response to the policy maker's choice of the precision of public information.

In particular, we assume that in period -1, after the state of the economy θ_{-1} in that period becomes common knowledge and after a (benevolent) planner chooses the precision of the public signal p_y, private agents simultaneously choose the precision of their private signals p_{x_i} about the period-0 fundamental θ. In period 0, after observing the signals (y, x_i), each agent i then chooses her action k_i. Actions are chosen simultaneously by the agents. For convenience, we summarize the timing of the model in Figure 1.

We denote by $C(p_{x_i})$ the cost of private information acquisition and assume that C is a continuously differentiable function satisfying $C'(p_x), C''(p_x) > 0$, all $p_x > 0$, $C'(0) = 0$ and

[6] The notation U_k denotes the partial derivative of U with respect to k, whereas the notation U_{kK} denotes the cross derivative with respect to k and K. Similar notation applies to the other arguments of the utility function.

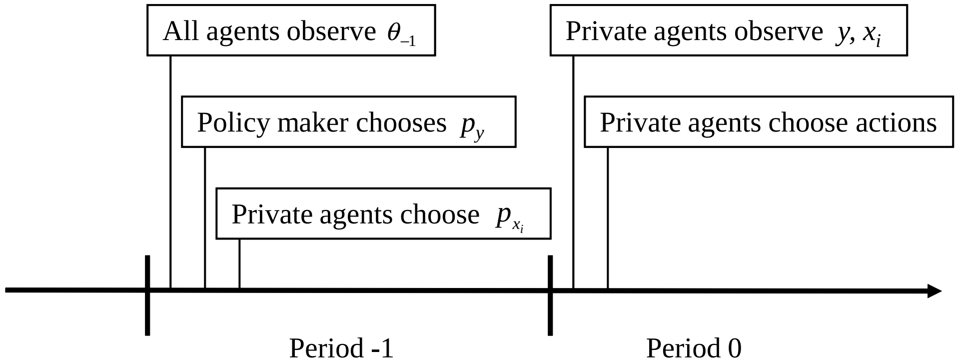

Figure 1: The timing of the model

$\lim_{p_x \to \infty} C'(p_x) = \infty$. The last two conditions are not crucial but guarantee interior solutions. The agent's payoff, net of the cost of information acquisition, is then given by $U(k_i, K, \sigma_k, \theta) - C(p_{x_i})$.

As standard in this literature, we do not model the cost for the provision of public information. The reason is that our interest is in the characterization of the inefficiencies in the collection of private information and on how they affect the marginal benefit of more precise public information. Introducing a cost for the provision of public information is necessary only if one wants to characterize the 'optimal' supply of public information. It is immediate to see how this can be done by combining our results with a specific cost function.

4 The equilibrium acquisition of private information

To solve for the equilibrium acquisition of private information, we start by revisiting how an agent's action k_i is affected by the quality of her private information p_{x_i}. These first steps follow closely from the analysis in Angeletos and Pavan (2007), adapted to the possibility that different agents possess information of different quality.

First note that, under complete information about θ, the unique equilibrium features each agent taking an action $k_i = \kappa(\theta)$ where $\kappa(\theta) \equiv \kappa_0 + \kappa_1(\theta)$ with $\kappa_0 \equiv \frac{-U_k(0,0,0,0)}{U_{kk}+U_{kK}}$ and $\kappa_1 \equiv \frac{-U_{k\theta}}{U_{kk}+U_{kK}}$.[7] Then consider the problem of an agent j, whose quality of information is p_{x_j}. Optimality requires that, for any (x, y), the agent's action $k_j = k_j(x, y; p_{x_j}, p_z)$ is such that

$$\mathbb{E}[U_k(k_j, \tilde{K}, \tilde{\sigma}, \tilde{\theta}) \mid x, y; p_{x_j}, p_z] = 0.$$

Using the fact that $U_k(\kappa, \kappa, 0, \theta) = 0$ and that U is quadratic, the above reduces to

$$k_j(x, y; p_{x_j}, p_z) = \mathbb{E}[(1-\alpha)\kappa(\tilde{\theta}) + \alpha\tilde{K} \mid x, y; p_{x_j}, p_z], \tag{1}$$

where $\alpha \equiv \frac{U_{kK}}{|U_{kk}|}$ measures the slope of individual best responses to aggregate activity, and is what in the literature is referred to as the 'equilibrium degree of coordination'.

[7] To save on notation, we will often replace $\kappa(\theta)$ with κ whenever there is no need to express the dependence on θ.

Now suppose that all agents $i \neq j$ acquired the same quality of information p_x. Because agent j has zero measure, from Proposition 1 in Angeletos and Pavan (2007), in the unique equilibrium of the continuation game, each agent $i \neq j$ follows the linear strategy $k_i = k(x, y; p_x, p_z)$ with

$$k(x, y; p_x, p_z) = \kappa_0 + \kappa_1 (\gamma z + (1 - \gamma) x), \tag{2}$$

where $z = \mathbb{E}\left[\tilde{\theta} \mid y\right] = \frac{p_\theta \theta_{-1} + p_y y}{p_\theta + p_y}$ and where $\gamma = \gamma(p_x, p_z)$ is given by

$$\gamma = \frac{\delta}{1 - \alpha(1 - \delta)} \text{ with } \delta \equiv \frac{p_z}{p_z + p_x}. \tag{3}$$

Then, the aggregate action is linear in (θ, y) and it can be written as

$$K(\theta, y; p_x, p_z) = \int_x k(x, y; p_x, p_z)\, dG(x \mid \theta, p_x) = \kappa_0 + \kappa_1 (\gamma z + (1 - \gamma)\theta),$$

where $G(x \mid \theta, p_x)$ denotes the cumulative distribution function of x conditional on θ when the precision of private information is p_x, and where $\gamma = \gamma(p_x, p_z)$ is as in (3). For future reference, we denote by $g(x|\theta, p_x)$ the density of $G(x|\theta, p_x)$.

Substituting $K = K(\theta, y; p_x, p_z)$ into (1), and using the fact that

$$\mathbb{E}\left[\tilde{\theta} \mid x_j, y; p_{x_j}, p_z\right] = \delta_j z + (1 - \delta_j) x_j,$$

where $\delta_j \equiv \frac{p_z}{p_z + p_{x_j}}$, we have that agent j's best response to all other agents following the strategy (2) is for her to follow the strategy

$$k_j\left(x, y; p_{x_j}, p_z\right) = \kappa_0 + \kappa_1 (\gamma_j z + (1 - \gamma_j) x), \tag{4}$$

where

$$\gamma_j = \frac{(1 - \alpha)\delta_j + \alpha\delta}{1 - \alpha(1 - \delta)}. \tag{5}$$

As it is evident from (5), the sensitivity of agent j's actions to the two sources of information is driven by (i) the quality of agent j's private information relative to that of public information, δ_j, (ii) the importance that she assigns to aligning her action to the other agents' actions, α, (iii) and the sensitivity of the other agents' actions to their private and public sources of information, which in turn depends on the 'commonality' of their information δ, that is, on the quality of other agents' private information p_x relative to public. As is well known, a stronger value α for aligning decisions induces more sensitivity to public sources of information, for the latter provide a better estimator of other agents' actions.

Knowing how the agent will use her information, we can now compute the quality of information that maximizes the agent's ex-ante utility as a best response to the quality of information p_x acquired by the other agents.

Let

$$\Pi(p_{x_j}, p_x, p_z) = \mathbb{E}[U(k_j(\tilde{x}_j, \tilde{y}; p_{x_j}, p_z), K(\tilde{\theta}, \tilde{y}; p_x, p_z), \sigma_k(\tilde{\theta}, \tilde{y}; p_x, p_z), \tilde{\theta}) \mid p_{x_j}, p_z] - C(p_{x_j})$$

denote the agent's ex-ante expected payoff when the quality of her private information is p_{x_j}, the quality of all other agents' private information is p_x, and the quality of public information is p_z. Note that the expectation is here over $(\tilde{\theta}, \tilde{y}, \tilde{x}_j)$. As we show in Appendix A (proof of Proposition 1), using the law of iterated expectations, integrating by parts, and applying the envelope theorem (which means disregarding the effects of a variation of p_{x_j} on k_j and simply focusing on how a variation of p_{x_j} impacts the distribution of x_j for each given θ), we have that

$$\begin{aligned}\frac{\partial \Pi(p_{x_j}, p_x, p_z)}{\partial p_{x_j}} &= \mathbb{E}\left[\begin{array}{c} U_k(k_j(\tilde{x}_j, \tilde{y}; p_{x_j}, p_z), K(\tilde{\theta}, \tilde{y}; p_x, p_z), \sigma_k(\tilde{\theta}, \tilde{y}; p_x, p_z), \tilde{\theta}) \cdot \\ \cdot \frac{\partial k_j(\tilde{x}_j, \tilde{y}; p_{x_j}, p_z)}{\partial x_j} \mathcal{I}(\tilde{x}_j, \tilde{\theta}; p_{x_j}) \mid p_{x_j}, p_z \end{array}\right] \\ &\quad - C'(p_{x_j}),\end{aligned} \tag{6}$$

where

$$\mathcal{I}(x_j, \theta; p_{x_j}) \equiv \frac{\frac{\partial \left[1 - G(x_j | \theta, p_{x_j})\right]}{\partial p_{x_j}}}{g(x_j | \theta, p_{x_j})}$$

is the information index corresponding to $(x_j, \theta; p_{x_j})$. This index captures how a variation in the precision p_{x_j} affects the probability of observing a signal above x_j in state θ, normalized by the density $g(x_j | \theta, p_{x_j})$.

The result in (6) is intuitive: the benefit of more precise private information comes from the fact that, on average, it allows the agent to increase her action in those situations where the marginal utility of a higher action is positive and reducing it in those situations where it is negative, thus increasing her payoff.

Using the normality of the information structure and the fact that the agent's optimal use of information is given by (4) yields

$$\frac{\partial \Pi(p_{x_j}, p_x, p_z)}{\partial p_{x_j}} = -\frac{\kappa_1(1-\gamma_j)}{2p_{x_j}} Cov\left[U_k(\tilde{k}_j, \tilde{K}, \sigma_k, \tilde{\theta}), (\tilde{x}_j - \tilde{\theta}) \mid p_{x_j}, p_x, p_z\right] - C'(p_{x_j}), \tag{7}$$

where $\tilde{k}_j = k_j(\tilde{x}_j, \tilde{y}; p_{x_j}, p_z)$, and $\tilde{K} = K(\tilde{\theta}, \tilde{y}; p_x, p_z)$. Using the fact that $U_k(k_j, K, \sigma_k, \theta) = U_k(\kappa, \kappa, 0, \theta) + U_{kk}(k_j - \kappa) + U_{kK}(K - \kappa)$, along with the fact that, by definition of the complete-information equilibrium, $U_k(\kappa, \kappa, 0, \theta) = 0$, and that the public signal, and thus the aggregate error $K - \kappa$, are orthogonal to the individual noise $\xi_j = x_j - \theta$, (7) reduces to

$$\frac{\partial \Pi(p_{x_j}, p_x, p_z)}{\partial p_{x_j}} = \frac{|U_{kk}|}{2} \frac{\kappa_1^2 (1-\gamma_j)^2}{\left(p_{x_j}\right)^2} - C'(p_{x_j}) \tag{8}$$

where $\gamma_j = \gamma_j(p_{x_j}, p_x, p_z)$ is given by (5). Because Π is concave in p_{x_j}, we have that the optimal choice of p_{x_j} is implicitly given by the first order condition

$$p_{x_j} = (1-\gamma_j)\sqrt{\frac{\kappa_1^2 |U_{kk}|}{2C'(p_{x_j})}}.$$

Using the definition of γ_j then yields the following result (proof in the Appendix).

Proposition 1 *Fix the precision of public information p_z and suppose that all agents $i \neq j$ acquire information of quality p_x. The best response of agent j is to acquire information of quality p_{x_j} implicitly defined by*

$$p_{x_j} + p_z = \frac{(1-\alpha)(p_x + p_z)}{(1-\alpha)p_x + p_z} \sqrt{\frac{\kappa_1^2 \, |U_{kk}|}{2C'(p_{x_j})}}. \tag{9}$$

A few observations are worth making. First, as one should expect, the amount of private information acquired by each individual is decreasing in the cost of information acquisition. Second, when actions are strategic complements (i.e., $\alpha \in (0,1)$), an increase in the precision of other agents' private information induces each agent to acquire more precise private information. The opposite is true when actions are strategic substitutes (i.e., when $\alpha \in (-1,0)$). This result is fully consistent with those of Hellwig and Veldkamp (2009) and Myatt and Wallace (2012) who note that, when actions are complements, agents want to know what others know, while the opposite is true when action are substitutes. The intuition for this result is simple. When other agents choose to acquire more precise private information, their private signals become more anchored around θ. If actions are strategic complements, agent j has an incentive to increase the precision of her private signal in order to better align it with the private information available to the other agents. Conversely, when actions are strategic substitutes, agent j aims at reducing the degree of alignment between her action and those of other agents, and to do so she reduces the precision of her private information.

One can also see from (9) that the precision of the information that each agent acquires is increasing in both (i) the sensitivity of the complete-information equilibrium actions κ_1 to fundamentals, and (ii) the curvature of individual payoffs, $|U_{kk}|$. Both effects should be expected, for they imply a higher value for aligning one's actions to the underlying fundamentals, and hence a higher benefit of more precise information.

Having characterized the properties of individual best responses, we now turn to the equilibrium collection of private information.

Proposition 2 *In the unique symmetric equilibrium, each agent acquires private information of precision $\hat{p}_x$ implicitly given by*

$$\hat{p}_x = \sqrt{\frac{|U_{kk}|\kappa_1^2}{2C'\left(\hat{p}_x\right)}} - \frac{p_z}{1-\alpha}. \tag{10}$$

The proof follows from setting $p_{x_j} = p_x$ in (9). Note that, in equilibrium, the dispersion of individual actions in the cross section of the population satisfies

$$Var[k(\tilde{x}, \tilde{y}; \hat{p}_x, p_z) - K(\tilde{\theta}, \tilde{y}; \hat{p}_x, p_z) \mid p_x, p_z] = \frac{\kappa_1^2(1-\gamma(\hat{p}_x, p_z))^2}{\hat{p}_x}.$$

Applying (8) to the symmetric case $\gamma_j = \gamma$, then note that, in equilibrium, the precision of private information $\hat{p}_x$ acquired by each agent is implicitly given by

$$-\frac{|U_{kk}|}{2}\frac{\partial Var[k(\tilde{x}, \tilde{y}; \hat{p}_x, p_z) - K(\tilde{\theta}, \tilde{y}; \hat{p}_x, p_z) \mid \hat{p}_x, p_z]}{\partial p_x} = \frac{|U_{kk}|}{2}\frac{\kappa_1^2(1-\gamma(\hat{p}_x, p_z))^2}{(\hat{p}_x)^2} = C'(\hat{p}_x), \tag{11}$$

where the partial derivative in (11) is obtained by holding constant both the distribution of the aggregate action $K(\theta, y)$ as well as the strategy $k(x, y|\hat{p}_x, p_z)$ that the individual plans to follow to map her information into her actions. In other words, in equilibrium, the marginal benefit that each agent assigns to an increase in the precision of her private information coincides with the marginal reduction in the dispersion of her action around the mean action, weighted by the importance $|U_{kk}|$ that the individual assigns to such reduction. This is intuitive, given that, from the usual envelope arguments, the individual expects her information to be used optimally once collected. As we will see below, this interpretation will help us understand the sources of inefficiency in the acquisition of private information. Note also that the amount of private information $\hat{p}_x$ collected in equilibrium is decreasing in the degree of strategic complementarity in agents' actions, α. This follows from the fact that a higher degree of strategic complementarity increases agents' incentives to align their actions, and hence it reduces the value they attach to learning the fundamental θ. The opposite result obtains under strategic substitutability.

Besides the comparative statics above, the most interesting feature of Condition (10) is that it permits us to characterize the effects of an increase in the precision of public information on the equilibrium acquisition of private information, which are summarized in the following Corollary, the proof of which follows directly from (10).

Corollary 1 (Crowding out effects of public information) *(i) An increase in the precision of public information reduces the precision of private information acquired in equilibrium:* $-\frac{1}{1-\alpha} \leq \frac{\partial \hat{p}_x}{\partial p_z} \leq 0$ *with* $\frac{\partial^2 \hat{p}_x}{(\partial p_z)^2} \geq 0$. *(ii) The substitutability between public and private information is increasing in the equilibrium degree of coordination:* $\frac{\partial^2 \hat{p}_x}{\partial \alpha \partial p_z} \leq 0$.

Part (i) of Corollary 1 highlights the substitutability between public and private information, while part (ii) highlights how such a substitutability increases with the equilibrium degree of coordination, α.

The intuition for part (i) is quite straightforward: when agents possess more precise public information, they can better forecast both the fundamental θ and the aggregate action K, in which case there is less value in acquiring private information. The intuition for part (ii) rests on the fact that the value of public information, relative to that of private information, consists in better permitting the agents to align their individual actions. The higher the value that the agents assign to aligning their actions, the higher the value of public information relative to private, and hence the stronger the substitutability between the two sources of information.[8] Finally, that the degree of substitutability between private and public information is decreasing in the precision of public

[8] The results in the Corollary are consistent with the findings in Propositions 1 and 2 of Myatt and Wallace (2012), who adopt a less general utility representation, but a more general information structure. A similar substitutability result can also be found in Wong (2008), who considers a setup in which an increase in the transparency of the information provided by a policy maker (i.e., a monetary authority) reduces the share of agents that purchase private information about the fundamental.

information comes from the convexity of the cost of acquiring private information: the higher the precision p_z of public information, the lower the precision $\hat{p}_x$ of private information in equilibrium, but then the lower the marginal cost of an increase in the precision of private information and hence the smaller the further reduction in the precision of private information triggered by an increase in the precision of public information.

5 The efficient acquisition of private information

We now turn to the characterization of the efficient acquisition of private information. Following the pertinent literature, the efficiency notion we use is the one corresponding to the team problem. In particular, we want to understand what is the best society could do if it could control the way its agents acquire and process information, but without being able to transfer information from one agent to another. As in the rest of the literature, the welfare criterion we adopt is the ex-ante utility of a representative agent, net of the cost of information acquisition:

$$\mathbb{E}[U(\tilde{k},\tilde{K},\tilde{\sigma}_k,\tilde{\theta}) \mid p_x,p_z] - C(p_x).$$

In other words, we are interested in a strategy $k^*(x,y)$ along with a precision of private information p_x^* that jointly maximize

$$\int_{(\theta,y)}\int_x U\left(k(x,y),K(\theta,y;p_x),\sigma_k(\theta,y;p_x),\theta\right) dG(x|\theta,p_x)dP(\theta,y;p_z) - C(p_x),$$

where $P(\theta,y;p_z)$ denotes the joint distribution of (θ,y) when the precision of public information is p_y, $K(\theta,y;p_x)=\int_x k\left(x,y\right)dG(x|\theta,p_x)$ denotes the level of aggregate activity when each agent follows the strategy $k\left(x,y\right)$, and $\sigma_k(\theta,y)=[\int_x[k\left(x,y\right)-K(\theta,y;p_x)]^2dP(x|\theta,p_x)]^{1/2}$ denotes the dispersion of individual actions in the cross section of the population. To this purpose, let

$$W(K,\sigma_k,\theta)\equiv U(K,K,\sigma_k,\theta)+\frac{1}{2}U_{kk}\sigma_k^2=\int U(k_i,K,\sigma_k,\theta)di$$

denote welfare under a utilitarian aggregator. Our interest is in allocations that maximize ex-ante utility. The function W is just a convenient instrument for computing ex-ante utility. Next, let $\kappa^*\left(\theta\right)$ be the unique solution to $W_K(\kappa^*,0,\theta)=0$; that is, $\kappa^*\left(\theta\right)=\kappa_0^*+\kappa_1^*\theta$, where $\kappa_0^*=-W_K\left(0,0,0\right)/W_{KK}$, $\kappa_1^*=-W_{K\theta}/W_{KK}$, $W_{KK}\equiv U_{kk}+2U_{kK}+U_{KK}<0$, and $W_{K\theta}\equiv U_{k\theta}+U_{K\theta}$. Note that $\kappa^*(\theta)$ is the first-best allocation. Following Angeletos and Pavan (2007), ex-ante utility for any arbitrary strategy $k(x,y)$ and arbitrary precisions of private and public information (p_x,p_y) can then be conveniently expressed as

$$\begin{aligned}\mathbb{E}[U(\tilde{k},\tilde{K},\tilde{\sigma}_k,\tilde{\theta}) \mid p_x,p_z] &= \mathbb{E}[W(\kappa^*(\tilde{\theta}),0,\tilde{\theta})] - \frac{|W_{KK}|}{2}\mathbb{E}[(K(\tilde{\theta},\tilde{y};p_x,p_z)-\kappa^*(\tilde{\theta}))^2 \mid p_x,p_z] \\ &\quad - \frac{|W_{\sigma\sigma}|}{2}\mathbb{E}[(k(\tilde{x},\tilde{y};p_x,p_z)-K(\tilde{\theta},\tilde{y};p_x,p_z))^2 \mid p_x,p_z],\end{aligned}$$

where $W_{\sigma\sigma} \equiv U_{kk} + U_{\sigma\sigma} < 0$. In other words, ex-ante utility under an arbitrary strategy $k(x, y)$ equals ex-ante utility under the first-best allocation κ^*, discounted by two losses.[9] The first one originates in the discrepancy between the aggregate activity K and the first-best activity. This inefficiency would obtain even if all agents were to take the same action. The second inefficiency originates in the dispersion of individual actions in the cross-section of the population.

Fixing the precision of private and public information, we also know from Angeletos and Pavan (2007) that the efficient use of information requires that all agents follow the linear strategy

$$k^* (x, y; p_x, p_z) = \kappa_0^* + \kappa_1^* (\gamma^* z + (1 - \gamma^*) x), \tag{12}$$

where $z = \mathbb{E}[\tilde{\theta} \mid y] = \frac{p_\theta \theta_{-1} + p_y y}{p_\theta + p_y}$,

$$\gamma^* = \frac{\delta}{1 - \alpha^* (1 - \delta)} \text{ with } \delta = \frac{p_z}{p_x + p_z}, \tag{13}$$

and

$$\alpha^* \equiv 1 - \frac{W_{KK}}{W_{\sigma\sigma}}. \tag{14}$$

Equivalently, the efficient strategy is the unique strategy that solves the functional equation

$$k (x, y; p_x, p_z) = \mathbb{E} \left[(1 - \alpha^*) \kappa^*(\tilde{\theta}) + \alpha^* K(\tilde{\theta}, \tilde{y}; p_x, p_z) \mid x, y; p_x, p_z \right] \text{ for all } (x, y), \tag{15}$$

where $K(\theta, y; p_x, p_z) = \int_x k(x, y; p_x, p_z) \, dG(x|\theta, p_x)$. The coefficient α^* can be interpreted as the socially optimal degree of coordination; it is the level of complementarity ($\alpha^* > 0$) or substitutability ($\alpha^* < 0$) that the planner would like all agents to perceive in order for the equilibrium of the economy to coincide with the efficient allocation.[10] From (14), one can verify that α^* is decreasing with social aversion to volatility ($|W_{KK}|$) and increasing with social aversion to dispersion ($|W_{\sigma\sigma}|$).

Note that, just as γ determines the relative sensitivity of equilibrium allocations to public and private information, γ^* determines the relative sensitivity of the efficient allocation to the two sources of information. By comparing γ to γ^*, it is therefore possible to see that the sensitivity of the equilibrium allocation to common noise is inefficiently large when and only when the equilibrium degree of coordination is higher than the socially optimal one; i.e., $\gamma \geq \gamma^*$ if and only if $\alpha \geq \alpha^*$.

Next note that, for any given precisions (p_x, p_z) of private and public information, welfare under the efficient allocation $k^*(x, y; p_x, p_z)$ can be expressed as

$$w^*(p_x, p_z) \equiv \mathbb{E}[W(\tilde{\kappa}^*, 0, \tilde{\theta})] - \mathcal{L}^*(p_x, p_z) - C(p_x),$$

where

$$\mathcal{L}^*(p_x, p_z) \equiv \frac{|W_{KK}|}{2} Var[\tilde{K}^* - \tilde{\kappa}^* \mid p_x, p_z] + \frac{|W_{\sigma\sigma}|}{2} Var[\tilde{k}^* - \tilde{K}^* \mid p_x, p_z],$$

[9] To save on notation, we let $\kappa^* = \kappa^*(\theta)$, whenever not confusing.

[10] The statement here is for economies that are efficient under complete information; i.e., for which $\kappa = \kappa^*$. As one can easily see from (15), efficiency in the use of information requires both that $\alpha = \alpha^*$ and that $\kappa = \kappa^*$.

and where $\tilde{\kappa}^*$, $\tilde{k}^*$, and $\tilde{K}^*$ are shortcuts for $\kappa^*(\tilde{\theta})$, $k^*(\tilde{x}, \tilde{y}; p_x, p_z)$ and

$$K^*(\tilde{\theta}, \tilde{y}; p_x, p_z) = \int_x k^*(x, \tilde{y}; p_x, p_z)\, dG(x|\tilde{\theta}, p_x).$$

We are now ready to turn to the efficient acquisition of private information. This is instrumental to our understanding of what inefficiency, if any, arises in the way information is collected in equilibrium, and on how such inefficiency relates to the way information is then used in equilibrium. As it will become clear in the next section, it is also instrumental to our understanding of the social value of public information, when one recognizes the endogenous response in the acquisition of private information.

Using the envelope theorem and observing that, holding constant the strategy k^*, the non-fundamental volatility of the efficient allocation is independent of the quality of private information, we then have that the efficient acquisition of private information p_x^* is given by the first-order condition

$$\frac{\partial w^*(p_x^*, p_z)}{\partial p_x} = -\frac{|W_{\sigma\sigma}|}{2} \frac{\partial Var[k^*(\tilde{x}, \tilde{y}; p_x^*, p_z) - K^*(\tilde{\theta}, \tilde{y}; p_x^*, p_z) \mid p_x^*, p_z]}{\partial p_x} - C'(p_x^*) = 0,$$

where $\partial Var[(k^* - K^*) \mid p_x^*, p_z]/\partial p_x$ is the partial derivative of dispersion with respect to the quality of private information, holding constant the efficient allocation.

Note that the social (marginal) benefit of more precise private information is simply the reduction in the dispersion of individual actions around the mean action, weighted by the social aversion to dispersion $|W_{\sigma\sigma}|/2$. Importantly, the marginal effect of an increase of p_x on dispersion is computed holding constant the strategy $k^*(x, y; p_x^*, p_z)$ that defines the efficient use of information.

Using (12), we then have that

$$Var[k^*(\tilde{x}, \tilde{y}; p_x, p_z) - K^*(\tilde{\theta}, \tilde{y}; p_x, p_z) \mid p_x, p_z] = \frac{(\kappa_1^*)^2 (1 - \gamma^*(p_x, p_z))^2}{p_x},$$

where $\gamma^* = \gamma^*(p_x, p_z)$ is given by (13). Hence, we can write

$$\frac{\partial w^*(p_x, p_z)}{\partial p_x} = \frac{|W_{\sigma\sigma}|}{2} \frac{(\kappa_1^*)^2 (1 - \gamma^*(p_x, p_z))^2}{p_x^2} - C'(p_x). \tag{16}$$

Using (13), we then obtain the following result, which follows directly from the arguments above along with the quasi-concavity of the welfare function $w^*(p_x, p_z)$.

Proposition 3 *The efficient acquisition of private information is for each agent to acquire private information of precision p_x^* implicitly given by*

$$p_x^* = \sqrt{\frac{|W_{\sigma\sigma}|\, (\kappa_1^*)^2}{2C'(p_x^*)}} - \frac{p_z}{1 - \alpha^*}.$$

Comparing the results in Propositions 2 and 3 then leads to the following conclusion, whose formal proof follows from comparing (11) and (16).

Proposition 4 *Let $\hat{p}_x$ denote the precision of private information collected in equilibrium and p_x^* the precision of private information that maximizes welfare when the planner can control the way agents use their available information. Then $\hat{p}_x < p_x^*$ (resp., $\hat{p}_x > p_x^*$) if and only if*

$$|U_{kk}|\kappa_1^2(1-\gamma(\hat{p}_x,p_z))^2 < |W_{\sigma\sigma}|\kappa_1^{*2}(1-\gamma^*(\hat{p}_x,p_z))^2 \tag{17}$$

(resp., if and only if the sign of the inequality in (17) is reversed).

To understand the result, recall from the analysis above that both the private and the social marginal benefit of an increase in the precision p_x of private information come from the marginal reduction in the cross-sectional dispersion of individual actions.[11] The magnitude of this reduction depends on the sensitivity of individual actions to private information, which is given by $\kappa_1^2(1-\gamma)^2$ under the equilibrium strategy and by $\kappa_1^{*2}(1-\gamma^*)^2$ under the efficient strategy. The weight that the planner assigns to reducing cross-sectional dispersion is $|W_{\sigma\sigma}|$, while the weight that the individual assigns to reducing the dispersion of her action around the mean action is $|U_{kk}|$. We thus have that the amount of private information collected in equilibrium falls short of the efficient level if and only if the marginal reduction in cross-sectional dispersion under the equilibrium allocation, weighted by the importance that each agent assigns to dispersion, falls short of the marginal reduction in dispersion under the efficient allocation, weighted by the importance that the planner assigns to dispersion. Put it differently, efficiency in the acquisition of information requires (i) efficiency in the use of information (formally, $\kappa_1(1-\gamma) = \kappa_1^*(1-\gamma^*)$) and (ii) alignment between the private and social benefit of reducing the dispersion of individual actions in the population, which obtains when and only when $|U_{kk}| = |U_{kk} + U_{\sigma\sigma}|$.[12]

The following corollary is then an immediate implication of the previous result.

Corollary 2 *(i) Consider economies that are efficient in their use of information ($\kappa = \kappa^*$ and $\alpha = \alpha^*$). Then $\hat{p}_x < p_x^*$ (resp., $\hat{p}_x > p_x^*$) if and only if $U_{\sigma\sigma} < 0$ (resp., if and only if $U_{\sigma\sigma} > 0$).*

(ii) Consider economies that are efficient under complete information and whose inefficiency in the collection of private information originates in the way information is used in equilibrium ($\kappa = \kappa^*$, $U_{\sigma\sigma} = 0$ but $\alpha \neq \alpha^*$). Then $\hat{p}_x < p_x^*$ (resp., $\hat{p}_x > p_x^*$) if and only if $\alpha > \alpha^*$(resp., if and only if $\alpha < \alpha^*$).

(iii) Consider economies whose inefficiency in the collection and use of information originates in the inefficiency of the complete-information allocation ($U_{\sigma\sigma} = 0$, $\alpha = \alpha^*$, but $\kappa \neq \kappa^*$). Then $\hat{p}_x < p_x^*$ (resp., $\hat{p}_x > p_x^*$) if and only if $\kappa_1 < \kappa_1^*$ (resp., if and only if $\kappa_1 > \kappa_1^*$).

[11] Both marginal reductions are computed holding constant, respectively, the equilibrium and the efficient strategies by usual envelope arguments.

[12] Note that, while in principle efficiency in the acquisition can obtain even without efficiency in the use of information, this can happen only under the knife-edge case where the discrepancy between $\kappa_1(1-\gamma)$ and $\kappa_1^*(1-\gamma^*)$ is perfectly offset by the discrepancy between $|U_{kk}|$ and $|W_{\sigma\sigma}|$.

Let's start with part (i). Because in these economies the equilibrium use of information is efficient, the marginal reduction in the cross-sectional dispersion of individual actions under the equilibrium strategy coincides with the marginal reduction under the efficient strategy. That the equilibrium use of information is efficient, however, does not guarantee that the private and the social marginal benefit of more precise private information coincide. The reason is that the private benefits fail to take into account the direct, non-strategic, effect that the dispersion of individual actions has on payoffs, as captured by $U_{\sigma\sigma}$. Because this externality has no strategic effects, it is not internalized and is thus a source of possible inefficiency in the collection of private information. In particular, the amount of private information collected in equilibrium falls short of the efficient level in the presence of a negative externality from dispersion, $U_{\sigma\sigma} < 0$, while the opposite is true for economies where the externality is positive.

Next, consider part (ii) and take an economy where $\alpha > \alpha^*$. Because there are no direct externalities from dispersion (i.e., $U_{\sigma\sigma} = 0$), the weight that private agents assign to a reduction in the cross-sectional dispersion of individual actions coincides with the socially optimal one (i.e., $|W_{\sigma\sigma}| = |U_{kk}|$). The discrepancy between the private and the social marginal benefit of an increase in the precision p_x of private information then simply comes from the fact that, in equilibrium, agents rely too little on private information when choosing their actions $(\kappa_1^2(1-\gamma))^2 < \kappa_1^{*2}(1-\gamma^*)^2)$. This implies that the marginal reduction in the cross-sectional dispersion of individual actions is more pronounced under the efficient strategy than under the equilibrium strategy. In turn, this makes the social marginal benefit of more precise private information larger than the private benefit, thus explaining the equilibrium underinvestment in the acquisition of private information.

The same logic explains part (iii) in the proposition with the discrepancy in the sensitivity of individual actions to private information now coming from the gap between the complete-information allocation, κ, and the first-best allocation, κ^*, as opposed to the gap between the equilibrium and the socially efficient degree of coordination.

The results in Proposition 4 compare the amount of private information collected in equilibrium with the amount that a planner would like the agents to collect, *if the planner could also dictate to the agents how to use their available information.* This comparison is of interest, for it tells us in which direction the policy maker would like to correct inefficiencies in the collection of private information when it can also correct inefficiencies in the use of information (see, e.g., Angeletos and Pavan, 2009, for how fiscal policy can restore efficiency in the use of information).

For certain problems of interest, though, it is important to compare the amount of private information collected in equilibrium with the amount that the planner would like the agents to collect *if the planner were unable to change the way society uses the information it collects.* This is akin to investigating how welfare, under the equilibrium allocation and net of the cost of information acquisition, changes with the precision of private information around the level $\hat{p}_x$ selected in equilibrium. As it will become clear in the next section, addressing this question is particularly relevant for the social value of public information; that is, for the comparative statics of welfare, under the equilibrium strategy, with respect to the quality of public information. The remainder of this section is thus devoted to the analysis of this question.

We start by noticing that, for any precisions (p_x, p_z) of private and public information, welfare under the equilibrium allocation is given by the same representation of equilibrium welfare following from Conditions (15) and (16) in Angeletos and Pavan (2007):

$$\begin{aligned} w(p_x, p_z) &\equiv \mathbb{E}[U(\tilde{k}, \tilde{K}, \tilde{\sigma}_k, \tilde{\theta}) \mid p_x, p_z] - C(p_x) \\ &= \mathbb{E}[W(\tilde{\kappa}, 0, \tilde{\theta})] - \mathcal{L}(p_x, p_z) - C(p_x), \end{aligned} \tag{18}$$

where $\mathbb{E}[W(\tilde{\kappa}, 0, \tilde{\theta})]$ is expected welfare under the complete-information allocation κ, whereas

$$\begin{aligned} \mathcal{L}(p_x, p_z) \equiv \frac{|W_{KK}|}{2} \cdot Var[\tilde{K} - \tilde{\kappa} \mid p_x, p_z] + \frac{|W_{\sigma\sigma}|}{2} \cdot Var[\tilde{k} - \tilde{K} \mid p_x, p_z] \\ - Cov\left[\tilde{K} - \tilde{\kappa}, W_K(\tilde{\kappa}, 0, \tilde{\theta}) \mid p_x, p_z\right] \end{aligned}$$

are the welfare losses due to incomplete information. The first two terms in $\mathcal{L}(p_x, p_z)$ are analogous to the two terms in $\mathcal{L}^*(p_x, p_z)$, except that they are computed under the equilibrium allocation, as opposed to the efficient allocation. They measure the welfare losses due to, respectively, the volatility of the aggregate action K around its complete-information counterpart κ, and the dispersion of individual actions around the aggregate action. The covariance term is a novel first-order effect that is present only in economies that are inefficient under complete information (i.e., for which $\kappa \neq \kappa^*$): a positive correlation between $K - \kappa$, the 'aggregate error' due to incomplete information, and W_K, the social return to aggregate activity, contributes to higher welfare, whereas a negative correlation between the two contributes to lower welfare. This covariance term can in turn be expressed as

$$Cov\left[\tilde{K} - \tilde{\kappa}, W_K(\tilde{\kappa}, 0, \tilde{\theta}) \mid p_x, p_z\right] = |W_{KK}| \cdot \phi \cdot v,$$

where

$$\phi \equiv \frac{Cov[\tilde{\kappa}, \tilde{\kappa}^* - \tilde{\kappa}]}{Var\,(\tilde{\kappa})} = \frac{\kappa_1^* - \kappa_1}{\kappa_1}$$

captures the covariance between the complete-information activity κ and the complete-information 'efficiency gap' $(\kappa^* - \kappa)$, whereas

$$v \equiv Cov[\tilde{K} - \tilde{\kappa}, \tilde{\kappa} \mid p_x, p_z] = -\frac{\kappa_1^2 \gamma(p_x, p_z)}{p_z} = -\frac{\kappa_1^2}{p_z + (1 - \alpha)\, p_x}$$

captures the covariance between the 'aggregate error' due to incomplete information $(K - \kappa)$ and the complete-information equilibrium (κ). Using the fact that

$$Var[\tilde{K} - \tilde{\kappa} \mid p_x, p_z] = \frac{\kappa_1^2 \gamma(p_x, p_z)^2}{p_z} = \frac{\kappa_1^2 p_z}{(p_z + (1 - \alpha)\, p_x)^2},$$

$$Var[\tilde{k} - \tilde{K} \mid p_x, p_z] = \frac{\kappa_1^2 (1 - \gamma(p_x, p_z))^2}{p_x} = \frac{\kappa_1^2\, (1 - \alpha)^2\, p_x}{(p_z + (1 - \alpha)\, p_x)^2},$$

and the definition of α^*, after some algebra, we then have that welfare under the equilibrium allocation can be expressed as

$$\begin{aligned} w(p_x, p_z) = {} & \mathbb{E}[W(\tilde{\kappa}, 0, \tilde{\theta})] + \\ & - \frac{|W_{KK}|}{2} \frac{\kappa_1^2 \gamma(p_x, p_z)^2}{p_z} - \frac{|W_{\sigma\sigma}|}{2} \frac{\kappa_1^2 (1 - \gamma(p_x, p_z))^2}{p_x} + \\ & - |W_{KK}| \left(\frac{\kappa_1^* - \kappa_1}{\kappa_1} \right) \frac{\kappa_1^2 \gamma(p_x, p_z)}{p_z} + \\ & - C(p_x). \end{aligned} \tag{19}$$

Using (19), we then have that the marginal effect on equilibrium welfare of an increase in the precision of private information p_x is given by[13]

$$\begin{aligned} \frac{\partial w(p_x, p_z)}{\partial p_x} = {} & \frac{|W_{\sigma\sigma}|}{2} \frac{\kappa_1^2 (1 - \gamma(p_x, p_z))^2}{(p_x)^2} - C'(p_x) \\ & + \frac{|W_{\sigma\sigma}| \kappa_1^2 (\alpha - \alpha^*)}{p_z + (1 - \alpha) p_x} \left| \frac{\partial \gamma(p_x, p_z)}{\partial p_x} \right| \\ & + \frac{|W_{KK}| \kappa_1^2}{p_z} \left(\frac{\kappa_1^* - \kappa_1}{\kappa_1} \right) \left| \frac{\partial \gamma(p_x, p_z)}{\partial p_x} \right| \end{aligned} \tag{20}$$

The first term in (20) is the direct marginal effect of a reduction in cross-sectional dispersion that obtains as a result of an increase in p_x, holding fixed the agents' strategy. The third term combines the marginal effects on volatility and dispersion of changing the agents' strategy by inducing them to rely more on their private information and less on their public information (recall that $\partial \gamma(p_x, p_z)/\partial p_x < 0$). Finally, the last term, which is relevant only in economies that are inefficient under complete information, captures the effect of changing the agents' strategy on $cov\left[\tilde{K} - \tilde{\kappa}, W_K(\tilde{\kappa}, 0, \tilde{\theta})\right]$, that is on the way the 'error' due to incomplete information covaries with the inefficiency of the complete-information allocation. Clearly, by usual envelope arguments, these last two terms are absent in economies where the equilibrium use of information is efficient ($\kappa = \kappa^*$ and $\alpha = \alpha^*$) or, alternatively, when the planner can dictate to the agents how to use their information.

Comparing the marginal benefit that each individual assigns in equilibrium to an increase in the precision p_x of her private information (as given by (11)) with the marginal benefit that the planner assigns to the same increase then yields the following result.

Proposition 5 *Let p_x^{**} denote the precision of private information that maximizes welfare when the planner cannot control the way the agents use their available information. The same conclusions as in parts (i)-(iii) of Corollary 2 hold relative to p_x^{**}.*

[13] The derivation here is different from that in Angeletos and Pavan (2007), reflecting the need to identify the sources of inefficiency in the acquisition of private information — see Appendix A for details.

Clearly, in economies that are efficient in their use of information ($\kappa = \kappa^*$ and $\alpha = \alpha^*$), the precision of private information that maximizes welfare is the same, irrespective of whether or not the planner can control the way the agents use their available information (i.e., $p_x^{**} = p_x^*$). As discussed above, in this case, inefficiencies in the equilibrium collection of private information originate entirely in the discrepancy between the private $|U_{kk}|$ and the social $|W_{\sigma\sigma}|$ weights assigned to reducing cross-sectional dispersion of individual actions.

Next, consider economies that are efficient under complete information and where the inefficiency in the collection of private information originates in the way information is used in equilibrium ($\kappa = \kappa^*$, $U_{\sigma\sigma} = 0$ but $\alpha \neq \alpha^*$). In this case, the amount of private information p_x^{**} that the planner would like the agents to collect when he cannot control the way the agents use their available information can be either higher or lower than the amount p_x^* that he would like them to collect when he can dictate to the agents how to use their available information. Nonetheless, compared to the amount $\hat{p}_x$ collected in equilibrium, the same conclusions hold as in part (ii) of Corollary 2. When agents are over-concerned about aligning their actions and hence rely too much on their public sources of information, the planner would like them to collect more precise private information than they do in equilibrium, so as to bring their use of information closer to the efficient level (recall that $\gamma(p_x, p_x)$ is decreasing in p_x). The opposite is true when the agents' concern about aligning their actions falls short of the efficient level, in which case the planner would like them to cut on their collection of private information (that is, $p_x^{**} < \hat{p}_x$ when $\alpha < \alpha^*$).

Finally, consider economies where the inefficiency in the collection and use of information originates in the inefficiency of the complete-information allocation ($U_{\sigma\sigma} = 0$, $\alpha = \alpha^*$, but $\kappa \neq \kappa^*$). In this case, the marginal effect on welfare of an increase in the precision of private information, evaluated at the equilibrium level $\hat{p}_x$, is given by

$$\frac{\partial w(\hat{p}_x, p_z)}{\partial p_x} = \frac{|W_{KK}|\kappa_1^2}{p_z} \left(\frac{\kappa_1^* - \kappa_1}{\kappa_1} \right) \left| \frac{\partial \gamma(\hat{p}_x, p_z)}{\partial p_x} \right|,$$

from which one can see that the amount of private information acquired in equilibrium is inefficiently low (resp. high) if $\kappa_1 < \kappa_1^*$ (resp., $\kappa_1 > \kappa_1^*$). The intuition for this result is simple. Economies where $\kappa_1 < \kappa_1^*$ are economies where

$$Cov \left[\tilde{K} - \tilde{\kappa}, W_K(\tilde{\kappa}, 0, \tilde{\theta}) \mid p_x, p_z \right] < 0.$$

That is, in these economies, the 'error' $\tilde{K} - \tilde{\kappa}$ due to incomplete information covaries negatively with the inefficiency $W_K(\kappa, 0, \theta)$ of the complete-information allocation, which brings the economy more far away from the first best level. More precise private information, by bringing the aggregate activity closer to the complete-information level then increases efficiency. This first-order effect is, however, not internalized by the agents, which explains why the planner would like them to collect more precise private information than they do in equilibrium, despite the planner's inability to control the way society uses the information it collects. The opposite is true for economies where the 'error' $\tilde{K} - \tilde{\kappa}$ covaries positively with the inefficiency of the complete-information allocation (that is, for economies where $\kappa_1 > \kappa_1^*$). In this case, 'ignorance is a blessing' for it partially corrects the inefficiency of the complete-information allocation. As a result, the planner would like society to collect less precise private information than it does in equilibrium.

Remark. The economies considered in Corollary 2 and Proposition 5 are benchmark cases where the source of the inefficiency in the collection of private information can be isolated and where the inefficiency can be unambiguously signed. More generally, one can show that there exists a function $\Lambda(\kappa_1^* - \kappa_1, \alpha - \alpha^*; p_z)$ that is increasing in the differences $(\kappa_1^* - \kappa_1, \alpha - \alpha^*)$ and equal to zero at $(0, 0)$, such that, for any $(\kappa_1^*, \kappa_1, \alpha, \alpha^*)$, the equilibrium precision of private information falls short of the efficient level; i.e., $\hat{p}_x < p_x^{**}$, if and only if $U_{\sigma\sigma} < \Lambda(\kappa_1^* - \kappa_1, \alpha - \alpha^*; p_z)$.

6 The social value of public information when private information is endogenous

We now turn to the welfare effects of variations in the precision of public information. The key novelty with respect to previous work in this literature is that the analysis below takes into account how variations in the precision of public information affect agents' incentives to acquire private information.

Intuitively, relative to the case where the precision of private information is exogenous, an increase in the precision of public information, by inducing agents to cut on their acquisition of private information (equivalently, by inducing them to reduce the attention they allocate to private sources of information), induces a stronger increase in the commonality $\delta = p_z/(p_x + p_z)$ of information and a smaller increase (or even a decrease) in the accuracy of information, $\sigma = p_z + p_x$. Whether the net effect on welfare is larger than when neglecting the endogeneity of private information in turn depends on whether the amount of private information collected in equilibrium is inefficiently low or high, as shown in the next proposition.[14]

Proposition 6 *Recognizing the crowding-out effects of public information on private information reduces the social value of public information if and only if the precision of the private information collected in equilibrium is inefficiently low:*

$$\frac{dw(\hat{p}_x, p_z)}{dp_z} < \frac{\partial w(\hat{p}_x, p_z)}{\partial p_z} \Leftrightarrow \hat{p}_x < p_x^{**}.$$

The result follows directly from the fact that

$$\frac{dw(\hat{p}_x, p_z)}{dp_z} = \frac{\partial w(\hat{p}_x, p_z)}{\partial p_z} + \frac{\partial w(\hat{p}_x, p_z)}{\partial p_x}\frac{\partial \hat{p}_x}{\partial p_z},$$

[14] As formally stated in the proposition, here the comparison is with respect to the precision of private information p_x^{**} that maximizes welfare when the planner can not control the way information is used in equilibrium.

along with the fact that a higher precision of public information always crowds out the acquisition of private information, as established in Corollary 1.

The following result is then an immediate implication of the above proposition along with the results in Corollary 2 and Proposition 5.

Corollary 3 *Recognizing the endogeneity of private information reduces the social value of public information in economies where* $\kappa_1 \leq \kappa_1^*$, $\alpha \geq \alpha^*$, *and* $U_{\sigma\sigma} \leq 0$ *(strictly when at least one of the inequalities is strict). It increases the social value of public information in economies where* $\kappa_1 \geq \kappa_1^*$, $\alpha \leq \alpha^*$, *and* $U_{\sigma\sigma} \geq 0$ *(again, strictly when at least one of the inequalities is strict).*

In certain cases, recognizing the endogeneity of private information is particularly important, for not only it affects the 'magnitude' of the social value of public information, it may even revert its sign (see the application to economies with negative production externalities in Section 7 below). However, as the next result shows, this is never the case in economies where the collection of private information is inefficiently low and where the equilibrium degree of coordination is not too high relative to the efficient one (the proof is in Appendix A).

Proposition 7 *Take any economy where the amount of private information collected in equilibrium is inefficiently low (i.e.,* $\hat{p}_x \leq p_x^{**}$*). There exists a critical threshold*

$$\Delta \equiv \frac{|W_{KK}| \cdot |U_{kk}|}{|W_{\sigma\sigma}| \cdot \{|W_{\sigma\sigma}| + |U_{kk}|\}}$$

such that, irrespective of the equilibrium degree of substitutability between public and private information ($d\hat{p}_x/dp_z$)*, welfare always increases with the precision of public information if* $\alpha - \alpha^* < \Delta$*; i.e., if the discrepancy between the equilibrium and the efficient degree of coordination is not too large.*

Note that, in economies where $\alpha - \alpha^* < \Delta$, both private and public information can contribute either positively or negatively to welfare; that is, $\partial w(\hat{p}_x, p_z)/\partial p_z$ and $\partial w(\hat{p}_x, p_z)/\partial p_x$ can be of either sign. What the bound on the discrepancy $\alpha - \alpha^*$ guarantees is that, whenever agents underinvest in their collection of private information (i.e., $\partial w(\hat{p}_x, p_z)/\partial p_x \geq 0$), then public information has a positive direct effect on welfare (i.e., $\partial w(\hat{p}_x, p_z)/\partial p_z \geq 0$). Furthermore, irrespective of the strength of the crowding-out effect that public information exerts on private information, the total effect of an increase in the precision of public information is always positive (i.e., $dw(\hat{p}_x, p_z)/dp_z \geq 0$).

The following corollary is then an implication of the previous proposition.

Corollary 4 *In economies where* $\alpha - \alpha^* < \Delta$*, recognizing the endogeneity of private information may either increase or decrease the social value of public information. However, it can never turn the social value of public information negative when it is positive ignoring the endogeneity of private information. That is,*

$$\frac{dw(\hat{p}_x, p_z)}{dp_z} \gtreqless \frac{\partial w(\hat{p}_x, p_z)}{\partial p_z} \text{ but } \frac{\partial w(\hat{p}_x, p_z)}{\partial p_z} \geq 0 \Rightarrow \frac{dw(\hat{p}_x, p_z)}{dp_z} \geq 0.$$

The first part of the statement follows directly from the fact that, in these economies, agents may either overinvest or underinvest in the acquisition of their private information. The crowding-out effect of public information on private information can thus either strengthen or weaken the direct effect that more precise public information exerts on welfare. The second part follows by contradiction. Suppose that the direct effect of an increase in the precision of public information on welfare is positive and yet the total effect (recognizing the endogenous response of private information) is negative. That is, suppose that

$$\frac{dw\left(\hat{p}_x, p_z\right)}{dp_z} < 0 \leq \frac{\partial w\left(\hat{p}_x, p_z\right)}{\partial p_z}.$$

For this to be possible, it must be that, at the equilibrium level, agents underinvest in their collection of private information, i.e., $\partial w\left(\hat{p}_x, p_z\right)/\partial p_x \geq 0$. But then the result in Proposition 7 implies that $dw(\hat{p}_x, p_z)/dp_z \geq 0$, a contradiction.

7 Applications

In this section, we show how the results can be put to work in a few applications of interest: beauty contests, monetary economies with price-setting complementarities, and economies with negative production externalities.

7.1 Beauty contests

Morris and Shin (2002) have shown that more precise public information may have a detrimental effect on welfare in economies that resemble Keynes' beauty contests. As it is by now well understood, the detrimental effect of public information stems from a coordination motive that is not warranted at the social level, namely from the fact that the equilibrium degree of coordination exceeds the socially optimal one, i.e., $\alpha > \alpha^*$. Such over-concern with coordination in turn induces agents to overreact to their public sources of information and underreact to their private ones, for the former are a better predictor than the latter of other agents' behavior.

The particular payoff specification used by Morris and Shin (2002) to illustrate this point is

$$U\left(k_i, K, \sigma_k, \theta\right) = -\left(1-r\right)\left(k_i - \theta\right)^2 - r\left(L_i - \bar{L}\right),$$

where $r \in (0,1)$ is a scalar that parametrizes the intensity of the coordination motive, $L_i = \int_{[0,1]} \left[k_h - k_i\right]^2 dh = \left(k_i - K\right)^2 + \sigma_k^2$, is the dispersion of other agents' actions around agent i's action, and $\bar{L} = \int_{[0,1]} L_i di = 2\sigma_k^2$ is a positive externality that comes from the dispersion of other agents' actions around the mean action. This payoff specification is thus nested in our model with $U_{kk} = -2$, $U_{kK} = 2r$, $U_{KK} = -2r$, $U_{k\theta} = 2\left(1-r\right)$, $U_{K\theta} = 0$, $U_{\sigma\sigma} = 2r$, $\alpha \equiv \frac{U_{kK}}{|U_{kk}|} = r$, $\kappa_1 \equiv -\frac{U_{k\theta}}{U_{kk}+U_{kK}} = \kappa_1^* \equiv -\frac{U_{K\theta}+U_{k\theta}}{U_{kk}+2U_{kK}+U_{KK}} = 1$, $W_{KK} \equiv U_{kk} + 2U_{kK} + U_{KK} = 2\left(r-1\right)$, $W_{\sigma\sigma} \equiv U_{kk} + U_{\sigma\sigma} = 2\left(r-1\right)$, and hence $\alpha^* \equiv 1 - \frac{W_{KK}}{W_{\sigma\sigma}} = 0$.

Combining (20) with (11), the marginal effect of an increase in the precision of private information, evaluated at the equilibrium level, is given by (see Appendix A for a formal derivation of the technical conditions in this application)

$$\frac{\partial w(\hat{p}_x, p_z)}{\partial p_x} = \frac{(1-r)^2\, r[p_z - (1-r)\,\hat{p}_x]}{(p_z + (1-r)\,\hat{p}_x)^3}. \tag{21}$$

As one can easily see from (21), the amount of private information collected in equilibrium can be either inefficiently low or inefficiently high, depending on the precision of public information p_z. Formally, there exists a threshold $p_z' > 0$ such that $\partial w(\hat{p}_x, p_z)/\partial p_x < 0$ if and only if $p_z < p_z'$.[15]

Likewise, the direct effect of an increase in the precision of public information on equilibrium welfare is given by

$$\frac{\partial w(\hat{p}_x, p_z)}{\partial p_z} = \frac{(1-r)\,[p_z - (2r-1)\,(1-r)\,\hat{p}_x]}{(p_z + (1-r)\,\hat{p}_x)^3}, \tag{22}$$

which is positive if and only if

$$p_z > (2r-1)(1-r)\hat{p}_x, \tag{23}$$

as shown in Morris and Shin (2002). Using the negative dependence of $\hat{p}_x$ on p_z one can then show that there exists a threshold $p_z'' < p_z'$ such that the inequality in (23) holds if and only if $p_z > p_z''$.

One can then easily see how recognizing the endogenous response in the acquisition of private information may affect the social value of public information. As shown in Proposition 6, when the precision of private information collected in equilibrium is inefficiently high (in these economies, this occurs when $p_z < p_z'$), recognizing the endogeneity of private information implies revising upwards the social value of public information. This new effect can be sufficiently strong to overturn the partial effect identified in the literature, making the social value of public information positive under the same conditions that would have predicted it to be negative by ignoring the endogeneity of private information.

To see this more clearly, suppose that the cost of private information acquisition is described by the iso-elastic cost function

$$C\,(p_x) = \frac{p_x^{1+\eta}}{1+\eta},$$

where $\eta \in [0, \infty)$ is the elasticity of the marginal cost. Using (10), one can check that the equilibrium degree of substitutability between public and private information (equivalently, the crowding-out effect of public information on private information) is given by

$$\frac{\partial \hat{p}_x}{\partial p_z} = -\frac{2\hat{p}_x}{\eta\,(p_z + (1-\alpha)\hat{p}_x) + 2\,(1-\alpha)\,\hat{p}_x}, \tag{24}$$

with

$$-\frac{1}{1-\alpha} \leq \frac{\partial \hat{p}_x}{\partial p_z} \leq 0,$$

[15] Observe from (10) that $\lim_{p_z \to 0} \hat{p}_x(p_z) = \bar{p}_x > 0$, with $\bar{p}_x$ implicitly given by $\bar{p}_x^2 C'(\bar{p}_x) = |U_{kk}|\kappa_1^2/2$, while $\lim_{p_z \to \infty} \hat{p}_x(p_z) = 0$. Along with the fact that $\hat{p}_x$ is strictly decreasing in p_z gives the result.

as predicted by Corollary 1. Combining (21) with (22) and using (24), one can then show that the total effect

$$\frac{dw(\hat{p}_x, p_z)}{dp_z} = \frac{\partial w(\hat{p}_x, p_z)}{\partial p_z} + \frac{\partial w(\hat{p}_x, p_z)}{\partial p_x}\frac{\partial \hat{p}_x}{\partial p_z}$$

of an increase in the precision of public information (when one recognizes the crowding-out effect of public information on private information) is positive if and only if

$$p_z \geq \left(2r - 1 - \frac{2\,(1-r)}{\eta}\right)(1-r)\hat{p}_x.$$

Given that

$$\left(2r - 1 - \frac{2\,(1-r)}{\eta}\right)(1-r)\hat{p}_x < (2r-1)(1-r)\hat{p}_x,$$

this means that there exists a third threshold $p_z''' < p_z''$ such that the social value of public information is positive, when recognizing the endogeneity of private information, if and only if $p_z > p_z'''$. It is then immediate that for any $p_z \in (p_z''', p_z'')$, the social value of public information turns from negative to positive when acknowledging the endogeneity of private information.

This result may seem counterintuitive at a first glance. Relative to the case in which the precision of private information is exogenous, an increase in the precision of public information induces a smaller increase in accuracy $\sigma^{-2} = p_z + \hat{p}_x$, and a larger increase in commonality $\delta = p_z/(p_z + \hat{p}_x)$. It is known that, in these economies, welfare increases with accuracy while it decreases with commonality, due to the inefficiently high level of coordination (i.e., to $\alpha > \alpha^*$).[16] When $p_z < (2r-1)(1-r)\hat{p}_x$ (equivalently, when $p_z < p_z''$), the positive welfare effects coming from the increase in accuracy are more than offset by the negative welfare effects due to the increase in commonality. Given that the endogeneity of private information reduces the positive effects of accuracy and increases the negative effects of commonality, how is it possible that recognizing the endogeneity of private information turns the social value of public information from negative to positive? The answer lies in the inefficiently high level of private information collected in equilibrium. By inducing agents to cut on their collection of private information, an increase in the precision of public information can boost welfare when the costs saved in the collection of private information more than compensate for the increase in non-fundamental volatility relative to dispersion.

At this point, one may wonder whether the opposite is also true: can the endogeneity of private information turn the sign of the social value of public information from positive to negative? As shown in Proposition 6, for this to be possible, it must be that the amount of private information collected in equilibrium is inefficiently low; i.e., $\hat{p}_x < p_x^{**}$. Recall that, in these economies, this occurs whenever $p_z > p_z'$ or, equivalently, whenever

$$p_z > (1-r)\,\hat{p}_x.$$

However, because, in these economies, the discrepancy between the equilibrium and the social degree of coordination $\alpha - \alpha^* = r$ is never larger than $\Delta = 1/(2-r)$, we know from Proposition

[16] See Corollary 8 in Angeletos and Pavan (2007).

(7) that the social value of public information always remains positive when it is positive ignoring the endogeneity of private information.

The following result summarizes the above observations.

Proposition 8 *Consider the class of 'beauty-contest' economies described above. Recognizing the crowding-out effects of public information on private information may either increase or decrease the social value of public information. However, while such effects may turn the sign of the social value of public information from negative to positive, they can never turn it from positive to negative.*

7.2 Monetary economies with price-setting complementarities

A recent literature investigates the welfare implications of information dispersion in monetary economies with price-setting complementarities (see, e.g., Hellwig, 2005, Adam, 2007, Roca, 2010, Lorenzoni, 2010, Angeletos, Iovino, and La'O, 2011). Here, we revisit the social value of public information in these economies, accounting for the endogeneity of the private information acquisition process. The description of how these economies can be traced back to the abstract specification of our model, after the usual linear-quadratic approximations, is relegated to Appendix B.

The economy is populated by a continuum of agents of measure one, indexed by $i \in [0,1]$. Each agent is both a consumer of all goods produced in the economy and the sole producer of good i, which is produced with labor as the only input. Each agent derives utility $J(C_i)$ from the consumption of the bundle of goods $(c_{hi})_{h\in[0,1]}$, where c_{hi} denotes the consumption of good $h \in [0,1]$ by agent i and where

$$C_i = \left(\int_{[0,1]} c_{hi}^{\frac{v-1}{v}} dh \right)^{\frac{v}{v-1}},$$

with $v > 1$ denoting the elasticity of substitution among goods. In addition, the agent suffers disutility $\theta V(Y_i)$ from producing Y_i units of good i. The functions J and V are both increasing and twice differentiable, with J concave and V convex. The parameter θ is a common shock, which characterizes the disutility of effort.

Each consumer-producer maximizes the (expectation of) the utility function

$$u(\theta, C_i, Y_i) \equiv J(C_i) - \theta V(Y_i)$$

subject to the budget constraint

$$\int_{[0,1]} p_h c_{hi} dh \leq (1+\tau) p_i Y_i - PT,$$

where τ denotes a subsidy optimally set by the fiscal authority to offset the usual distortions due to monopolistic competition, P is the Dixit-Stiglitz price index, and T is the lump-sum tax in real terms used to finance the subsidy.

The timing is such that each agent i chooses the price p_i for the good she produces before learning the shock θ. She then commits to supply any quantity that is demanded in equilibrium at that price. As is standard in this literature, we assume that consumption decisions occur under complete information. Before setting their prices, agents choose the precision of their private information in response to the precision p_z of public information set by the government.

The description of the model is completed by assuming that the influence of monetary policy on output is summarized by the quantity equation $\int_{[0,1]} p_h Y_h dh = \bar{M}$, which can also be interpreted as a simple nominal GDP targeting; here Y_h is aggregate demand for good h, while $\bar{M}$ is the aggregate money supply.

Note that this setup is similar to those in Hellwig (2005), Adam (2007), and Lorenzoni (2010), except for the fact that each consumer i is the sole producer of good i, which simplifies the analysis.[17]

As we show in Appendix B, after the usual linear-quadratic approximations, this framework can be traced back to the abstract reduced-form specification of our model by letting k_i denote the price set by agent i (normalized by the non-stochastic steady state price level), K the aggregate price, and σ_k^2 the dispersion of individual prices in the cross section of the population. Each consumer-producer's *indirect* utility – gross of the costs of information acquisition – can then be expressed as follows

$$U\left(k_i, K, \sigma_k, \theta\right) \cong \bar{U}\left(k_i, K, \theta\right) + \frac{\Psi}{2}\left\{-v\sigma_k^2 + \right. \\ \left. -\omega\left[\left(v-1\right)K - vk_i\right]^2 - 2\theta\left[\left(v-1\right)K - vk_i\right]\right\}, \tag{25}$$

where the function $\bar{U}\left(k_i, K, \theta\right)$ collects all terms that are constant or linear in $\left(k_i, K, \theta\right)$, $\Psi \equiv J_C\left(\bar{Y}\right)\bar{Y}$ with $\bar{Y}$ denoting the non-stochastic equilibrium level of output and consumption,[18] and

$$\omega \equiv -\frac{\left(J_{CC}\left(\bar{Y}\right) - V_{YY}\left(\bar{Y}\right)\right)\bar{Y}}{J_C\left(\bar{Y}\right)} > 0$$

captures both the curvature of the marginal utility of consumption and the sensitivity of producers' prices to the output gap.

It is then easy to see that, in these economies, $U_{kk} = -\Psi\omega v^2$, $U_{kK} = \Psi\omega v\left(v-1\right)$, $U_{KK} = -\Psi\omega\left(v-1\right)^2$, $U_{k\theta} = \Psi v$, $U_{K\theta} = -\Psi(v-1)$, and $U_{\sigma\sigma} = -\Psi v < 0$. Thus, this is an economy where actions (prices) are strategic complements, with a degree of equilibrium coordination given by $\alpha \equiv \frac{U_{kK}}{|U_{kk}|} = \frac{v-1}{v} > 0$ that depends on the elasticity of substitution v among goods.[19] An increase

[17] Another difference is that Hellwig (2005) assumes that agents are uncertain about monetary policy shocks but do not face idiosyncratic utility shocks. Hence, an increase in information transparency reduces price dispersion in his setup. Conversely, Lorenzoni (2010) focuses on disaggregated shocks. In his setup, more precise public information increases price dispersion, which however raises welfare since it also helps agents setting relative prices more aligned to productivity differentials.

[18] The non-stochastic equilibrium is defined to be the equilibrium that obtains when the variance σ_θ^2 of the labor supply shock θ is equal to 0 and by normalizing $\theta_{-1} = 1$.

[19] In Roca (2010), the degree of complementarity in prices is affected both by v and by ω, which is the parameter summarizing the curvature of the utility function. Conversely, in Adam (2007) and Baeriswyl and Cornand (2007), the parameter v does not play any role, as in these papers individual prices are directly proportional to the aggregate price level.

in the price level K raises the demand for the product supplied by agent i, which in turn induces her to increase her own price k_i. Note also that this economy features a negative externality from price dispersion; i.e., $U_{\sigma\sigma} < 0$. Observing that $W_{KK} \equiv U_{kk} + 2U_{kK} + U_{KK} = -\Psi\omega$ and $W_{\sigma\sigma} \equiv U_{kk} + U_{\sigma\sigma} = -\Psi v(1 + \omega v)$, we then have that the socially optimal degree of coordination is $\alpha^* \equiv 1 - \frac{W_{KK}}{W_{\sigma\sigma}} = \frac{(v^2-1)\omega+v}{v(1+\omega v)} > \alpha$. The reason why the equilibrium degree of coordination falls short of the socially optimal one is that each price setter i disregards the contribution of her price to σ_k^2, that is, to the dispersion of individual prices around the mean price.

Finally, notice that $\kappa_1 \equiv -\frac{U_{k\theta}}{U_{kk}+U_{kK}} = \kappa_1^* \equiv -\frac{W_{K\theta}}{W_{KK}} = 1/\omega$, implying that under perfect information, the price setters' reaction to a change in the fundamental is first-best efficient. This should not surprise given that the reduced-form payoff in (25) is computed under the subsidy that eliminates the distortions due to imperfect competition. However, since $U_{\sigma\sigma} < 0$ and $\alpha < \alpha^*$, these economies are inefficient both in the acquisition and in the use of private information.

Using Condition (20), one can then show that the marginal effect on welfare of an increase in the precision of private information, evaluated at the equilibrium level $\hat{p}_x$, is given by:[20]

$$\frac{\partial w(\hat{p}_x, p_z)}{\partial p_x} = \frac{\Psi v}{2\left(\omega\bar{\theta}\right)^2 (vp_z + \hat{p}_x)^3} \left(\hat{p}_x - v(2\omega(v-1)+1)p_z\right). \tag{26}$$

Accordingly, there exists a critical threshold p_z' such that price setters acquire too little private information when $p_z < p_z'$ (in which case $\hat{p}_x > v(2\omega(v-1)+1)p_z$) and too much private information when $p_z > p_z'$ (in which case $\hat{p}_x < v(2\omega(v-1)+1)p_z$).

Turning to the effects of public information, by differentiating (19), we have that the direct effect on welfare of an increase in p_z (holding constant $\hat{p}_x$) is given by

$$\frac{\partial w(\hat{p}_x, p_z)}{\partial p_z} = \frac{\Psi\omega v^2}{2\left(\omega\bar{\theta}\right)^2 (vp_z + \hat{p}_x)^2} + \frac{\Psi v^2\left(\omega(v-1)+1\right)\hat{p}_x}{\left(\omega\bar{\theta}\right)^2 (vp_z + \hat{p}_x)^3}. \tag{27}$$

That the direct effect of public information on welfare is always positive follows from the fact that, in these economies, welfare increases with both accuracy and commonality (since $\kappa_1 = \kappa_1^*$ and $\alpha < \alpha^*$).[21]

As for the total effect of an increase in p_z on welfare, note that, because $\alpha - \alpha^* < \Delta$ and because the direct effect $\partial w(\hat{p}_x, p_z)/\partial p_z$ is always positive, from Proposition (7) we have that the total effect $dw(\hat{p}_x, p_z)/dp_z$ is also always positive. Thus, we can state the following result.

Proposition 9 *Consider the class of economies with price-setting complementarities described above. Recognizing the crowding-out effects of public information on the acquisition of private information may either increase or decrease the social value of public information. However, the latter is always positive in these economies.*

[20] The formulas in (26) and (27) below can also be obtained by applying Conditions (32) and (33) in the proof of Proposition 7 in the Appendix to the payoff structure considered in this application.

[21] See Proposition 7 in Angeletos and Pavan (2007).

7.3 Negative production externalities

Lastly, consider the following stylized consumer-producer competitive economy. There is a continuum of identical agents. Each agent produces k_i units of a non-differentiated private good using labor as the only input. Her utility function, gross of the cost of private information acquisition, is given by

$$U(k_i, K, \sigma_k, \theta) = \theta J(k_i) - V(k_i, K).$$

The term $\theta J(k_i)$ captures the utility from consuming the good, with θ denoting an aggregate taste shock, while the term $V(k_i, K)$ captures the disutility of producing the good. We assume that J is increasing, differentiable, and concave and that V is increasing, twice differentiable, and convex in each argument; i.e., $V_k(k_i, K) > 0$, $V_{kk}(k_i, K) \geq 0$, $V_K(k_i, K) > 0$, and $V_{KK}(k_i, K) \geq 0$. Finally, we assume that the marginal disutility of producing the good increases with the aggregate activity K; i.e., $V_{kK}(k_i, K) > 0$. The negative effect exerted by aggregate production K on both $U(k_i, K, \sigma_k, \theta)$ and $U_k(k_i, K, \sigma_k, \theta)$ captures negative externalities such as those due to pollution or congestion.[22]

By taking a quadratic expansion of $U(k_i, K, \sigma_k, \theta)$ around the non-stochastic equilibrium $\bar{k}_i = \bar{K}$ corresponding to $\theta = \theta_{-1}$ (recall that θ_{-1} is the unconditional mean for θ) and normalizing $\theta_{-1} = 1$, we can conveniently write $U(k_i, K, \sigma_k, \theta)$ as follows

$$U(k_i, K, \sigma_k, \theta) = \bar{U}(k_i, K, \theta) + \bar{J}_k k_i \theta + \frac{1}{2} k_i^2 \left(\bar{J}_{kk} - \bar{V}_{kk}\right) - \bar{V}_{kK} k_i K - \frac{\bar{V}_{KK}}{2} K^2,$$

where $\bar{U}(k_i, K, \theta)$ collects all the terms that are constant or linear in (k_i, K, θ) and where $\bar{J}_k \equiv J_k(\bar{K}) > 0$, $\bar{J}_{kk} \equiv J_{kk}(\bar{K}) < 0$, $\bar{V}_{kk} \equiv V_{kk}(\bar{K}, \bar{K}) > 0$, $\bar{V}_{kK} \equiv V_{kK}(\bar{K}, \bar{K}) > 0$, and $\bar{V}_{KK} \equiv V_{KK}(\bar{K}, \bar{K}) > 0$. Letting $\omega \equiv -\frac{(\bar{J}_{kk} - \bar{V}_{kk})\bar{K}}{\bar{J}_k} > 0$ and $\chi \equiv \bar{V}_{KK}/\bar{V}_{kK} > 0$, we can write

$$\begin{aligned} U(k_i, K, \sigma_k, \theta) = {} & \bar{U}(k_i, K, \theta) - \left(\bar{J}_{kk} - \bar{V}_{kk}\right)\left(\frac{1}{\omega}\bar{K} k_i \theta - \frac{1}{2} k_i^2\right) + \\ & - \bar{V}_{kK}\left(k_i K + \frac{\chi}{2} K^2\right). \end{aligned}$$

We then have that $U_{kk} = \bar{J}_{kk} - \bar{V}_{kk} < 0$, $U_{kK} = -\bar{V}_{kK} < 0$, $U_{KK} = \chi U_{kK}$, $U_{k\theta} = -\frac{(\bar{J}_{kk} - \bar{V}_{kk})\bar{K}}{\omega} > 0$. Since $U_{\sigma\sigma} = 0$, this application features no direct externalities from dispersion. Contrary to the previous two applications, the equilibrium degree of strategic complementarity $\alpha \equiv \frac{U_{kK}}{|U_{kk}|} = \frac{\bar{V}_{kK}}{\bar{J}_{kk} - \bar{V}_{kk}} < 0$, reflecting the negative marginal externality on production coming from high aggregate activity.

Furthermore, $\kappa_1 \equiv -\frac{U_{k\theta}}{U_{kk} + U_{kK}} = \frac{\bar{K}}{\omega(1-\alpha)} > 0$ while $W_{KK} \equiv U_{kk} + 2U_{kK} + U_{KK} = (1 - (2 + \chi)\alpha) U_{kk} < 0$ and $W_{K\theta} \equiv U_{k\theta} + U_{K\theta} = U_{k\theta} = -\frac{(\bar{J}_{kk} - \bar{V}_{kk})\bar{K}}{\omega} > 0$ so that the first-best response to a change

[22] Classical examples include Tybout (1972) and Rothenberg (1970). Models with similar qualitative features also obtain in different frameworks, such as those related to the exploitation of natural resources (e.g., Scott Gordon, 1954, and Baumol and Oates, 1988), or to the private provision of public goods (e.g., Bergstrom, Blume and Varian, 1986).

in the fundamental is $\kappa_1^* \equiv -\frac{W_{K\theta}}{W_{KK}} = \frac{\bar{K}}{\omega(1-\alpha(2+\chi))} > 0$. Hence, in the complete-information equilibrium, agents overreact to a change in the fundamental; that is, $\kappa_1 > \kappa_1^* > 0$.

From $W_{\sigma\sigma} \equiv U_{kk} + U_{\sigma\sigma} = U_{kk}$ we then have that $\alpha^* \equiv 1 - \frac{W_{KK}}{W_{\sigma\sigma}} = (\chi+2)\,\alpha < \alpha$, implying that the socially optimal degree of coordination falls short of the privately perceived one. This follows directly from the fact that individual producers ignore their contribution to the negative externality.

Hence, this economy is inefficient in the use of information. To see how such inefficiency in turn impacts the efficiency in the collection of private information and ultimately the social value of public information, note that, after some algebra, (20) is equal to[23]

$$\frac{\partial w\left(\hat{p}_x, p_z\right)}{\partial p_x} = \frac{\alpha\left(1+\chi\right)|U_{kk}|\bar{K}^2\hat{p}_x}{\omega^2\left(p_z+(1-\alpha)\hat{p}_x\right)^3} < 0, \tag{28}$$

meaning that, in equilibrium, agents acquire too much private information relative to what is socially efficient.

Turning to the social value of public information, using (19), we have

$$\frac{\partial w\left(\hat{p}_x, p_z\right)}{\partial p_z} = \frac{|U_{kk}|\bar{K}^2[(1+\chi\alpha)\,p_z + (1-\alpha)\left(1+2\alpha+3\alpha\chi\right)\hat{p}_x]}{2\omega^2\left(1-\alpha\right)^2\left(p_z+(1-\alpha)\hat{p}_x\right)^3} \tag{29}$$

that, together with (28), yields

$$\frac{dw\left(\hat{p}_x, p_z\right)}{dp_z} =$$
$$= \frac{|U_{kk}|\bar{K}^2}{\omega^2\left(p_z+(1-\alpha)\hat{p}_x\right)^3}\left(\frac{(1+\chi\alpha)\,p_z + (1-\alpha)\left(1+2\alpha+3\alpha\chi\right)\hat{p}_x}{2\left(1-\alpha\right)^2} + \alpha\left(1+\chi\right)\hat{p}_x\frac{\partial\hat{p}_x}{\partial p_z}\right).$$

From Proposition 6, it is then immediate that, in these economies, recognizing the endogeneity of private information contributes to a higher social value of public information (thanks to its crowding out effect on the acquisition of private information, which is always inefficiently high). Whether the total effect is positive or negative depends on the strength of the degree of substitutability α perceived in equilibrium. When $\alpha < -1/\chi$, the negative direct effect that more precise public information has on welfare is so strong that, irrespective of the substitutability between public and private information (recall that $\frac{\partial\hat{p}_x}{\partial p_z} \geq -\frac{1}{1-\alpha}$ from Corollary 1), the total effect of an increase in p_z on welfare is always negative, so that it is always optimal to provide the economy with as little public information as possible. Likewise, when $-1/(2+3\chi) \leq \alpha < 0$, the direct positive effect that public information has on welfare is so strong that welfare would increase with the precision of public information, even if one were to ignore the endogeneity of private information.

Finally, consider the more interesting case in which $-1/\chi < \alpha < -1/(2+3\chi)$. Recognizing the endogeneity of private information may then turn the sign of the social value of public information

[23]The formulas in (28) and (29) below can also be obtained by applying Conditions (32) and (33) in the proof of Proposition 7 in the Appendix to the payoff structure considered in this application.

from negative to positive. Recalling that $\lim_{p_z \to 0} \hat{p}_x = \bar{p}_x > 0$ and $\lim_{p_z \to \infty} \hat{p}_x = 0$, there exists a critical level p'_z such that, in the absence of any substitutability between private and public information, welfare increases with p_z if and only if $p_z \geq p'_z$. By continuity, we then have that there exists a second critical threshold $p''_z < p'_z$ such that, for any $p_z \in (p''_z, p'_z)$

$$\frac{\partial w(\hat{p}_x, p_z)}{\partial p_z} < 0 < \frac{dw(\hat{p}_x, p_z)}{dp_z}.$$

That is, the social value of public information is negative when ignoring the endogeneity of private information and positive otherwise.

We summarize the above observations in the following proposition.

Proposition 10 *Consider the class of economies with negative production externalities described above, featuring an excessively low degree of strategic substitutability in actions, $\alpha^* < \alpha < 0$, and an excessively high sensitivity of the complete-information allocation to the underlying fundamental, $\kappa_1 > \kappa_1^*$. Recognizing the crowding-out effects of public information on private information always increases the social value of public information, turning the latter positive in situations where it would have been negative ignoring such effects.*

8 Concluding remarks

This paper investigated the sources of inefficiency in the equilibrium acquisition of private information. It then showed how the social value of public information is affected by the endogenous response in the acquisition of private information.

In future work, it would be interesting to extend the analysis to richer specifications of the information structure, such as those considered in the rational inattention literature as well as those recently studied in Myatt and Wallace (2012). That paper allows agents to access at a cost a variety of signals whose 'publicity' (i.e., correlation of errors across agents) is determined endogenously by the attention paid by the agents to the different sources of information. Paralleling the analysis in the current paper, it would be interesting to examine what drives inefficiencies in the attention that agents allocate to the different sources of information. Such a characterization could in turn be used to study how welfare in these economies is affected by variations in the quality of the different sources of information (in the terminology of Myatt and Wallace, in the quality of 'sender noise') taking into account the agents' endogenous response in their allocation of attention (that is, the endogeneity of 'receiver noise').

Another promising line of research consists in exploring the implications of our results in fully micro-founded models of the business cycle such as those recently considered in Angeletos, Iovino and La'O (2011) and in Paciello and Wiederholt (2011).

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Appendix A. Proofs

Proof of Proposition 1. Agent j's best response to the other agents collecting information of quality p_x when the quality of public information is p_z is to collect information of quality p_{x_j} so as to maximize

$$\Pi(p_{x_j}, p_x, p_z) = \mathbb{E}[U(k_j(\tilde{x}_j, \tilde{y}; p_{x_j}, p_z), K(\tilde{\theta}, \tilde{y}; p_x, p_z), \sigma_k(\tilde{\theta}, \tilde{y}; p_x, p_z), \tilde{\theta}) \mid p_{x_j}, p_z] - C(p_{x_j})$$

Using the law of iterated expectations, integrating by parts, and applying the envelope theorem (which means disregarding the effects of a variation of p_{x_j} on k_j and simply focusing on how a variation of p_{x_j} impacts the distribution $G(x_j|\theta, p_{x_j})$ for each given θ), the agent's first order condition is

$$\frac{\partial \Pi(p_{x_j}, p_x, p_z)}{\partial p_{x_j}} = -\int_{\theta,y} \left[\begin{array}{c} \int_{x_j} U_k(k_j(x_j, \tilde{y}; p_{x_j}), K(\theta, y; p_x, p_z), \sigma_k(\theta, y; p_x, p_z), \theta) \cdot \\ \frac{\partial k_j(x_j, y; p_{x_j})}{\partial x_j} \frac{\partial G(x_j|\theta, p_{x_j})}{\partial p_{x_j}} dx_j \end{array} \right] dP(\theta, y|p_z) \\ - C'(p_{x_j})$$

that, using the definition of $\mathcal{I}(x_j, \theta; p_{x_j})$, rewrites as (6) in the main text. Using the normality of the information structure

$$\frac{\partial G\left(x_j \mid \theta, p_{x_j}\right)}{\partial p_{x_j}} = \frac{\phi(\sqrt{p_{x_j}}(x_j - \theta))(x_j - \theta)}{2\sqrt{p_{x_j}}} = \frac{\sqrt{p_{x_j}}\phi(\sqrt{p_{x_j}}(x_j - \theta))(x_j - \theta)}{2p_{x_j}} = \frac{g(x_j|\theta, p_{x_j})(x_j - \theta)}{2p_{x_j}},$$

where $\phi(\cdot)$ is the density of the Standard Normal distribution. This means that

$$\mathcal{I}(x_j, \theta; p_{x_j}) \equiv \frac{\frac{\partial\left[1 - G\left(x_j|\theta, p_{x_j}\right)\right]}{\partial p_{x_j}}}{g(x_j|\theta, p_{x_j})} = -\frac{x_j - \theta}{2p_{x_j}}. \tag{30}$$

Replacing (30) into (6) and using the fact that

$$\frac{\partial k_j(x_j, y; p_{x_j})}{\partial x_j} = \kappa_1(1 - \gamma_j),$$

yields (7). The rest of the proof follows from the arguments in the main text. Q.E.D.

Derivation of Condition (20). Using (19), we have that

$$\begin{aligned} \frac{\partial w(p_x, p_z)}{\partial p_x} = & \frac{|W_{\sigma\sigma}|}{2} \frac{\kappa_1^2(1 - \gamma(p_x, p_z))^2}{(p_x)^2} - C'(p_x) + \\ & - \frac{|W_{KK}|}{2} \frac{\kappa_1^2}{p_z} \frac{\partial[\gamma(p_x, p_z)]^2}{\partial p_x} - \frac{|W_{\sigma\sigma}|}{2} \frac{\kappa_1^2}{p_x} \frac{\partial[1 - \gamma(p_x, p_z)]^2}{\partial p_x} + \\ & - |W_{KK}| \left(\frac{\kappa_1^* - \kappa_1}{\kappa_1} \right) \frac{\kappa_1^2}{p_z} \frac{\partial \gamma(p_x, p_z)}{\partial p_x}. \end{aligned}$$

Substituting $|W_{KK}| = (1-\alpha^*)|W_{\sigma\sigma}|$, the sum of the third and fourth addendum can be rewritten as

$$\left\{\frac{(1-\gamma(p_x,p_z))}{p_x} - \frac{(1-\alpha^*)\gamma(p_x,p_z)}{p_z}\right\}|W_{\sigma\sigma}|\kappa_1^2\frac{\partial\gamma(p_x,p_z)}{\partial p_x}. \tag{31}$$

Using the fact that

$$\frac{1-\gamma^*(p_x,p_z)}{p_x} - \frac{(1-\alpha^*)\gamma^*(p_x,p_z)}{p_z} = 0,$$

we then have that (31) is equivalent to

$$(\gamma^*(p_x,p_z) - \gamma(p_x,p_z))\left\{\frac{1}{p_x} + \frac{(1-\alpha^*)}{p_z}\right\}|W_{\sigma\sigma}|\kappa_1^2\frac{\partial\gamma(p_x,p_z)}{\partial p_x}.$$

Using the definitions of $\gamma^*(p_x,p_z)$ and $\gamma(p_x,p_z)$ then gives the result. Q.E.D.

Proof of Proposition 7. Replacing the formula for $\gamma(p_x,p_z)$ into (20) and using (11), we have that the marginal effect of an increase in p_x on equilibrium welfare, evaluated at the equilibrium precision of private information $\hat{p}_x$ is given by.

$$\begin{aligned}\frac{\partial w(\hat{p}_x,p_z)}{\partial p_x} &= \left(\frac{|W_{\sigma\sigma}|}{2} - \frac{|U_{kk}|}{2}\right)\frac{\kappa_1^2(1-\alpha)^2}{(p_z+(1-\alpha)\hat{p}_x)^2} + \\ &+ \frac{|W_{\sigma\sigma}|\kappa_1^2(\alpha-\alpha^*)(1-\alpha)p_z}{(p_z+(1-\alpha)\hat{p}_x)^3} + \\ &+ \frac{|W_{KK}|\kappa_1^2(1-\alpha)}{(p_z+(1-\alpha)\hat{p}_x)^2}\left(\frac{\kappa_1^*-\kappa_1}{\kappa_1}\right).\end{aligned} \tag{32}$$

Likewise, using (19), after some algebra, we have that the direct effect of an increase in the precision of public information p_z on welfare is given by

$$\begin{aligned}\frac{\partial w(\hat{p}_x,p_z)}{\partial p_z} &= \frac{|W_{KK}|}{2}\frac{\kappa_1^{*2}}{(p_z+(1-\alpha)\hat{p}_x)^2} + \\ &- \frac{|W_{\sigma\sigma}|}{2}\kappa_1^2(\alpha-\alpha^*)(1-\alpha)\left(\frac{2\hat{p}_x}{(p_z+(1-\alpha)\hat{p}_x)^3}\right) + \\ &- \frac{|W_{KK}|}{2}\frac{(\kappa_1^*-\kappa_1)^2}{(p_z+(1-\alpha)\hat{p}_x)^2}.\end{aligned} \tag{33}$$

Combining (32) with (33) and using the result in Corollary 1 that

$$\frac{\partial\hat{p}_x}{\partial p_z} \geq -\frac{1}{1-\alpha},$$

we have that, when $\partial w(\hat{p}_x,p_z)/\partial p_x > 0$,

$$\begin{aligned}\frac{dw(\hat{p}_x,p_z)}{dp_z} &= \frac{\partial w(\hat{p}_x,p_z)}{\partial p_z} + \frac{\partial w(\hat{p}_x,p_z)}{\partial p_x}\frac{\partial\hat{p}_x}{\partial p_z} \\ &\geq \frac{\partial w(\hat{p}_x,p_z)}{\partial p_z} - \frac{\partial w(\hat{p}_x,p_z)}{\partial p_x}\frac{1}{1-\alpha} \\ &= \frac{\kappa_1^2}{2(p_z+(1-\alpha)\hat{p}_x)^2}\{-|W_{\sigma\sigma}|(\alpha-\alpha^*) + |U_{kk}|(1-\alpha)\} \\ &= \frac{\kappa_1^2}{2(p_z+(1-\alpha)\hat{p}_x)^2}\left\{-(|W_{\sigma\sigma}|+|U_{kk}|)(\alpha-\alpha^*) + |U_{kk}|\frac{|W_{KK}|}{|W_{\sigma\sigma}|}\right\},\end{aligned}$$

which is always positive for $\alpha - \alpha^* < \Delta$. Q.E.D.

Derivation of the results for the 'beauty-contests' application. Conditions (21) and (22) follow from specializing Conditions (32) and (33) in the proof of Proposition 7 to the payoff structure considered in this application. Combining (21) with (22) and using (24), we have that

$$\frac{dw(\hat{p}_x, p_z)}{dp_z} = \frac{\partial w(\hat{p}_x, p_z)}{\partial p_z} + \frac{\partial w(\hat{p}_x, p_z)}{\partial p_x} \frac{\partial \hat{p}_x}{\partial p_z}$$

$$= \frac{(1-r)}{(p_z + (1-r)\hat{p}_x)^3} \left(p_z - (2r-1)(1-r)\hat{p}_x - \frac{r(1-r)(p_z - (1-r)\hat{p}_x)\, 2\hat{p}_x}{\eta\,(p_z + (1-r)\hat{p}_x) + 2\,(1-r)\,\hat{p}_x} \right).$$

Hence, $dw(\hat{p}_x, p_z)/dp_z \geq 0$ whenever

$$\eta p_z^2 + 2(1-r)^2(1+\eta)\hat{p}_x p_z - (\eta\,(2r-1) - 2\,(1-r))\,(1-r)^2\hat{p}_x^2 \geq 0,$$

or equivalently whenever

$$p_z \geq (1-r)\left(2r - 1 - 2\frac{(1-r)}{\eta}\right)\hat{p}_x,$$

as claimed in the main text. Q.E.D.

Appendix B. Price setting complementarities

In this appendix, we show how the economy described in Section 7.2 can be traced back to our linear-quadratic specification, after the usual approximations.

We start by characterizing the demand for each good, given the observed prices. In equilibrium, each agent i chooses her consumption bundle so as to maximize

$$u(C_i, Y_i, \theta) \equiv J\,(C_i) - \theta V\,(Y_i)$$

subject to the budget constraint

$$\int_{[0,1]} p_h c_{hi} dh \leq p_i Y_i(1+\tau) - PT,$$

where Y_i denotes the total demand for the product produced by agent i, and

$$P \equiv \left(\int_{[0,1]} p_h^{1-v} dh \right)^{\frac{1}{1-v}} \tag{34}$$

denotes the Dixit-Stiglitz price index. At the optimum, the budget constraint binds and each agent i's demand for each good h is given by

$$c_{hi} = \left(\frac{P}{p_h}\right)^v C_i. \tag{35}$$

Aggregating the individual demand functions (35) for each good h over all agents i, we then obtain that the aggregate demand for each good h is given by

$$Y_h = \left(\frac{P}{p_h}\right)^v Y, \tag{36}$$

where

$$Y \equiv \int_{[0,1]} C_i di.$$

Furthermore, substituting the individual demand for each good h, as given by (35), into the budget constraint, and using the definition of the aggregate price index in (34), we have that each agent's budget constraint can be rewritten as

$$C_i = (1+\tau)\frac{p_i}{P}Y_i - T. \tag{37}$$

By substituting the aggregate demand for good i, as given by (36), into the budget constraint (37) and the latter back into the individual demand for good h (i.e., into Condition (35)), we obtain the following optimality condition

$$c_{hi} = \left(\frac{P}{p_h}\right)^v \left((1+\tau)\left(\frac{P}{p_i}\right)^{v-1} Y - T\right). \tag{38}$$

Using (38), we then arrive to the following first-order approximation around the non-stochastic steady state corresponding to the complete-information equilibrium for $\theta = \theta_{-1}$ (in the steady state, $c_{hi} = \bar{C} = \bar{Y} = Y_i$, all $i, h \in [0,1]$, $p_h = p_i = P = \bar{P}$, $Y = \bar{Y}$, and $T = \bar{T}$):

$$\begin{aligned} c_{hi} \cong\ & \bar{Y} + v\left(P - \bar{P}\right)\frac{\bar{Y}}{\bar{P}} - v\left(p_h - \bar{P}\right)\frac{\bar{Y}}{\bar{P}} + \\ & + (1+\tau)(v-1)(P-\bar{P})\frac{\bar{Y}}{\bar{P}} + \\ & - (1+\tau)(v-1)(p_i-\bar{P})\frac{\bar{Y}}{\bar{P}} + (1+\tau)\left(Y-\bar{Y}\right) - \left(T-\bar{T}\right). \end{aligned}$$

Given that the production subsidy that offsets the distortion from monopolistic competition is $\tau = \frac{1}{v-1}$, and taking into account that $\bar{T} = \tau\bar{Y}$ and that $T = \tau Y$, we obtain that[24]

$$c_{hi} - \bar{Y} \cong v\left(P - p_h\right)\frac{\bar{Y}}{\bar{P}} + v\left(P - p_i\right)\frac{\bar{Y}}{\bar{P}} + Y - \bar{Y}. \tag{39}$$

Hence, the aggregate consumption $\tilde{c}_h = \int_{[0,1]} c_{hi} di$ of good h in the cross section of the population is approximated by

$$\tilde{c}_h - \bar{Y} \cong v\left(P - p_h\right)\frac{\bar{Y}}{\bar{P}} + v\left(P - \tilde{p}\right)\frac{\bar{Y}}{\bar{P}} + Y - \bar{Y}, \tag{40}$$

[24] To see why these two conditions hold, note that, from the optimality condition (38), the non-stochastic steady-state consumption levels must satisfy $\bar{C} = (1+\tau)\bar{Y} - \bar{T}$. Together with $\bar{C} = \bar{Y}$, this implies that $\bar{T} = \tau\bar{Y}$. Likewise, using (35), note that $Y = \frac{(1+\tau)}{P}\int_{[0,1]} p_i Y_i di - T$, which together with the fact that $\int_{[0,1]} p_i Y_i = PY$ (following from (36)) along with the definition of P and Y, implies that $T = \tau Y$.

where $\tilde{p} \equiv \int_{[0,1]} p_i di$ denotes the average price.

By subtracting (40) from (39), we obtain that

$$\frac{c_{hi} - \tilde{c}_h}{\bar{Y}} = v \frac{(\tilde{p} - p_i)}{\bar{P}}.$$

Letting $\sigma_h^2 \equiv \int_{[0,1]} \left(\frac{c_{hi} - \tilde{c}_h}{\bar{C}} \right)^2 di$ denote the cross sectional variance in the consumption of good h, normalized by the non-stochastic steady state level of aggregate consumption, denoting by $\sigma_p^2 \equiv \int_{[0,1]} \left(\frac{p_i - \tilde{p}}{\bar{P}} \right)^2 di$ the cross sectional dispersion of prices, normalized by the non-stochastic steady state price level, and using the fact that $\bar{Y} = \bar{C}$, we then have that

$$\sigma_h^2 = v^2 \sigma_p^2.$$

Next, we take a second-order expansion of C_i around $\bar{C}$ that gives

$$\begin{aligned} C_i = \bar{C} &+ \left(\int_{[0,1]} c_{hi}^{\frac{v-1}{v}} \Big|_{\bar{C}} dh \right)^{\frac{1}{v-1}} \left(\int_{[0,1]} c_{hi}^{-\frac{1}{v}} \Big|_{\bar{C}} \left(c_{hi} - \bar{C} \right) dh \right) + \\ &+ \frac{1}{2v} \left(\int_{[0,1]} c_{hi}^{\frac{v-1}{v}} \Big|_{\bar{C}} dh \right)^{\frac{2-v}{v-1}} \left(\int_{[0,1]} c_{hi}^{-\frac{1}{v}} \Big|_{\bar{C}} (c_{hi} - \bar{C}) dh \right)^2 + \\ &- \frac{1}{2v} \left(\int_{[0,1]} c_{hi}^{\frac{v-1}{v}} \Big|_{\bar{C}} dh \right)^{\frac{1}{v-1}} \int_{[0,1]} c_{hi}^{-\frac{1+v}{v}} \Big|_{\bar{C}} \left(c_{hi} - \bar{C} \right)^2 dh, \end{aligned}$$

which reduces to

$$C_i = \bar{C} + \int_{[0,1]} \left(c_{hi} - \bar{C} \right) dh + \frac{1}{2v\bar{C}} \left(\left(\int_{[0,1]} \left(c_{hi} - \bar{C} \right) dh \right)^2 - \int_{[0,1]} \left(c_{hi} - \bar{C} \right)^2 dh \right).$$

Using $\tilde{c}_i \equiv \int_{[0,1]} c_{hi} dh$, the approximation for C_i becomes

$$C_i - \bar{C} = \tilde{c}_i - \bar{C} + \frac{1}{2v\bar{C}} \left[\left(\tilde{c}_i - \bar{C} \right)^2 - \int_{[0,1]} \left(c_{hi} - \tilde{c}_i + \tilde{c}_i - \bar{C} \right)^2 dh \right],$$

or, equivalently,

$$\begin{aligned} C_i - \bar{C} = \tilde{c}_i - \bar{C} &+ \frac{1}{2v\bar{C}} \left(\tilde{c}_i - \bar{C} \right)^2 + \\ &- \frac{1}{2v\bar{C}} \left(\int_{[0,1]} (c_{hi} - \tilde{c}_i)^2 dh + \left(\tilde{c}_i - \bar{C} \right)^2 + 2 \left(\tilde{c}_i - \bar{C} \right) \int_{[0,1]} \left(c_{hi} - \tilde{c}_i \right) dh \right). \end{aligned}$$

Recalling the definition of $\tilde{c}_i$, this reduces to

$$C_i - \bar{C} = \tilde{c}_i - \bar{C} - \bar{C} \frac{\sigma_i^2}{2v}, \tag{41}$$

where $\sigma_i^2 \equiv \int_{[0,1]} \left(\frac{c_{hi} - \tilde{c}_i}{\bar{C}} \right)^2 dh$.

Now note that, when applied to good i, Condition (40) becomes

$$\tilde{c}_i - \bar{Y} \cong v\left(P - p_i\right)\frac{\bar{Y}}{\bar{P}} + v\left(P - \tilde{p}\right)\frac{\bar{Y}}{\bar{P}} + Y - \bar{Y}, \tag{42}$$

Substituting (42) and $\sigma_i^2 = v^2\sigma_p^2$, into (41) and using $\bar{Y} = \bar{C}$, we obtain that

$$C_i - \bar{C} \cong v\left(P - p_i\right)\frac{\bar{Y}}{\bar{P}} + v\left(P - \tilde{p}\right)\frac{\bar{Y}}{\bar{P}} + Y - \bar{Y} - \bar{Y}\frac{v\sigma_p^2}{2}. \tag{43}$$

By applying (39) to good i and agent j, we have that agent j's individual demand for good i is

$$c_{ij} - \bar{Y} \cong v\left(P - p_i\right)\frac{\bar{Y}}{\bar{P}} + v\left(P - p_j\right)\frac{\bar{Y}}{\bar{P}} + Y - \bar{Y}.$$

Aggregating across all agents j and using $Y_i = \int_{[0,1]} c_{ij} dj$ yields

$$Y_i - \bar{Y} \cong v\left(P - p_i\right)\frac{\bar{Y}}{\bar{P}} + v\left(P - \tilde{p}\right)\frac{\bar{Y}}{\bar{P}} + Y - \bar{Y}. \tag{44}$$

Lastly, using (34) and (36), note that the money supply equation can be rewritten as $PY = \bar{M}$. The first-order Taylor approximation for the money market equilibrium is

$$\left(P - \bar{P}\right)\bar{Y} + \bar{P}\left(Y - \bar{Y}\right) = 0,$$

so that we obtain

$$Y - \bar{Y} = -\left(P - \bar{P}\right)\bar{Y}/\bar{P}. \tag{45}$$

By using (45) and the fact that $P = \tilde{p}$ (by the first-order approximation of (34)), conditions (43) and (44) become

$$C_i - \bar{C} \cong \left[v\left(\tilde{p} - p_i\right) - \left(\tilde{p} - \bar{P}\right)\right]\frac{\bar{Y}}{\bar{P}} - \bar{Y}\frac{v\sigma_p^2}{2} \tag{46}$$

and

$$Y_i - \bar{Y} \cong \left[v\left(\tilde{p} - p_i\right) - \left(\tilde{p} - \bar{P}\right)\right]\frac{\bar{Y}}{\bar{P}}, \tag{47}$$

respectively.

Lastly, following the pertinent literature, we compute a second-order approximation of each agent's utility function around the non-stochastic steady state:

$$\begin{aligned} J\left(C_i\right) - \theta V\left(Y_i\right) \cong\ & J\left(\bar{C}\right) - \theta_{-1} V\left(\bar{Y}\right) + J_C\left(\bar{C}\right)\left(C_i - \bar{C}\right) - \theta_{-1} V_Y\left(\bar{Y}\right)\left(Y_i - \bar{Y}\right) + \\ & - V\left(\bar{Y}\right)\left(\theta - \theta_{-1}\right) + \frac{J_{CC}\left(\bar{C}\right)}{2}\left(C_i - \bar{C}\right)^2 - \frac{\theta_{-1} V_{YY}\left(\bar{Y}\right)}{2}\left(Y_i - \bar{Y}\right)^2 + \\ & - V_Y\left(\bar{Y}\right)\frac{\left(\theta - \theta_{-1}\right)}{2}\left(Y_i - \bar{Y}\right). \end{aligned} \tag{48}$$

Now observe that, due to the effect of the production subsidy $\tau = \frac{1}{v-1}$, the non-stochastic steady state corresponding to the complete-information equilibrium for $\theta = \theta_{-1}$ satisfies

$$J_C\left(\bar{Y}\right) = \theta_{-1} V_Y\left(\bar{Y}\right). \tag{49}$$

Substituting (46) and (47) into (48) and using (49), we obtain that

$$J(C_i) - \theta V(Y_i) \cong J(\bar{Y}) - \theta_{-1} V(\bar{Y}) - J_C(\bar{Y}) \frac{v}{2}\sigma_p^2 \bar{Y} - V(\bar{Y})(\theta - \theta_{-1}) +$$
$$+ \frac{J_{CC}(\bar{Y}) - \theta_{-1} V_{YY}(\bar{Y})}{2} \left((v-1)\frac{\tilde{p}}{\bar{P}} - v\frac{p_i}{\bar{P}} + 1\right)^2 \bar{Y}^2 +$$
$$- J_C(\bar{Y}) \frac{\theta - \theta_{-1}}{\theta_{-1}} \left((v-1)\frac{\tilde{p}}{\bar{P}} - v\frac{p_i}{\bar{P}} + 1\right) \bar{Y} +$$
$$- \frac{J_{CC}(\bar{Y})}{2} \left(\left((v-1)\frac{\tilde{p}}{\bar{P}} - v\frac{p_i}{\bar{P}} + 1\right) \bar{Y}^2 v \sigma_p^2\right) + \frac{J_{CC}(\bar{Y})}{2} \left(\bar{Y}\frac{v}{2}\sigma_p^2\right)^2.$$

By collecting in $\bar{U}\left(\theta, \frac{\tilde{p}}{\bar{P}}, \frac{p_i}{\bar{P}}\right)$ all terms that are constant or linear in $(\theta, \tilde{p}, p_i)$, letting $\theta_{-1} = 1$, $\omega \equiv -\frac{\left(J_{CC}(\bar{Y}) - \theta_{-1} V_{YY}(\bar{Y})\right)\bar{Y}}{J_C(\bar{Y})} > 0$, and $\Psi \equiv J_C(\bar{Y})\bar{Y}$, and disregarding all terms of order higher than two then leads to

$$J(C_i) - \theta V(Y_i) \cong \bar{U}\left(\theta, \frac{\tilde{p}}{\bar{P}}, \frac{p_i}{\bar{P}}\right) + \frac{\Psi}{2}\left(-v\sigma_p^2 - \omega\left((v-1)\frac{\tilde{p}}{\bar{P}} - v\frac{p_i}{\bar{P}}\right)^2 + \right.$$
$$\left. - 2\theta\left((v-1)\frac{\tilde{p}}{\bar{P}} - v\frac{p_i}{\bar{P}}\right)\right). \tag{50}$$

Replacing $k_i \equiv p_i/\bar{P}$, $K \equiv \tilde{p}/\bar{P}$, and $\sigma_k^2 = \sigma_p^2$, we then have that (50) reduces to (25) in the main text. Q.E.D.